Estimating default barriers from market information *

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Abstract

Brockman and Turtle (2003) develop a barrier option framework to show that default barriers are significantly positive. Most implied barriers are typically larger than the book value of corporate liabilities. We show theoretically and empirically that this result is biased due to the approximation of the market value of corporate assets by the sum of the market value of equity and the book value of liabilities. This approximation leads to a significant overestimation of the default barrier. To get rid of this bias, we propose a maximum likelihood (ML) estimation approach to estimate the asset values, asset volatilities, and default barriers. The proposed framework is applied to empirically examine the default barriers of a large sample of industrial firms. This paper documents that default barriers are positive but not very significant. In our sample, most of the estimated barriers are lower than the book values of corporate liabilities. In addition to the problem with the default barriers, we find significant biases on the estimation of asset value and asset volatility by Brockman and Turtle (2003).

JEL classification: G12; G33

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1 Introduction

The option-theoretic approach to corporate security valuation originated from the seminal works of Black and Scholes (1973) and Merton (1973, 1974). Based on this framework, equity value is viewed as a standard call (SC) option of corporate assets with a strike price that is equal to the promised payment of corporate liabilities. However, the conventional view of equity value as an SC option is inadequate to describe the consequences of bankruptcy at all times before an option's maturity. The work of Black and Cox (1976) supplements the Black-Scholes-Merton (BSM) framework by imposing a failure barrier (default barrier) to trigger bankruptcy before the maturity. When the underlying asset price breaches the barrier, corporate equity can be knocked out by bankruptcy so that bond holders are able to receive the remaining value of the firm before it deteriorates further. Black and Cox (1976) stress that the default barrier is relevant to bond protective covenants. As a result, corporate equity is modeled as a down-and-out call (DOC) option, and corporate debt is valued as a portfolio of default-free debt, a short put option, and a long down-and-in call.

These insights have had a profound impact both on financial theory and practice. With the concept of the default barrier, theoretical work has been carried out on the debt valuation and optimal capital structure (see, for example, Longstaff and Schwartz, 1995; Leland and Toft, 1996; Brisys and de Varenne, 1997). The option-theoretic approach also facilitates parameter estimation and empirical analysis for corporate bond pricing models and credit risk models. For instance, Moody's KMV Corporation estimates the value of the assets and volatilities of firms using a barrier option framework in which the default barrier is set as the default point, which is short-term debt plus half of the long-term debt (Crosbe and Bohn, 1993). Eom, Helwege and Huang (2003) test various structural models of corporate bond pricing by specifying the default barriers to be the book value of corporate liabilities.

Instead of specifying default barriers subjectively, we propose a statistical framework to estimate their value. Our approach greatly improves the barrier option framework of Brockman and Turtle (2003). In their paper, the market value of corporate assets is approximated by the market value of equity plus the book value of corporate liabilities (hereafter the proxy). After substituting the proxy into the DOC option pricing formula, a default barrier is extracted by setting the option price to be equal to the market value of equity. They then set up a hypothesis test to show that barriers, that are obtained in this way are statistically significant for a large cross-section of industrial firms. Their robustness tests reveal that "implied" barriers remain significant over a wide range of input parameters. However, we discover that using the proxy effectively overstates the default barrier. Therefore, the Brockman and Turtle (2003) framework indirectly assigns positive barriers to each firm before the statistical tests. Any results that are obtained in this way are unable to reflect the actual information that is contained in market variables.

This paper applies economic theories to uncover the implications of using the proxy. It is interesting and important in its own right because there has been much academic research that applies the same approximation, some examples being the work of Ogden (1987), Jung, Kim and Stulz (1996), Barclay and Smith (1997a, b), Lyden and Saraniti (2000), and Eom, Helwege and Huang (2003). We are concerned about the possible errors and presumptions that are behind the proxy. Using the properties of the SC and DOC options, we prove that implied barriers are larger than the book value of corporate liabilities if the proxy is adopted. This claim is true irrespective of the empirical data that are observed. The consequence is that unbiased empirical analysis for default barriers should not use the proxy.

The significant bias that is introduced by the proxy motivates us to investigate an alternative framework for default barrier estimation. We consider a maximum likelihood (ML) estimation, as it is an asymptotically unbiased approach. With a large enough sample, maximum likelihood estimators are close to true parameter values. Like Black and Cox (1976) and Brockman and Turtle (2003), we view the corporate equity as a DOC option of corporate assets, but instead of using the proxy, we construct an ML estimation of the default barrier, the underlying asset values, and the asset volatility. Previous work on ML estimation concentrates on spot prices and volatility (see, for example, Duan, 1994; Ericsson and Reneby, 2002), but our approach extends it to include the default barrier estimate. Because of the asymptotic unbias property of the ML estimation, our estimated parameters serve as a benchmark to assess the quality of the Brockman and Turtle (2003) approach.

This paper contributes to the literature by theoretically deriving the implications of using the proxy, by empirically examining the significance of default barriers, and by proposing a statistical framework to capture the effect of barriers on corporate claims. We estimate the default barriers, the market values of the assets of firms, and asset volatilities for a sample of 13,317 firm-years. We find that most estimated barriers are positive, but are less than the book value of corporate liabilities. Moreover, the default barriers that are implied by the market are typically high for the year of financial distress. We also highlight other problems of the framework of Brockman and Turtle (2003) besides the problem of the bias in default barrier estimates. We find that the firm asset values and asset volatilities that are obtained by this framework contain tremendous bias.

The rest of the paper is organized as follows. Section 2 derives the implications of using the proxy using financial arguments. Section 3 develops an ML estimation of the default barrier. We verify the proposed framework by simulation. Empirical results are presented in Section 4, and conclusive remarks are made in the final section.

2 Implications of the proxy

A typical example of a default barrier estimation that uses the proxy is the recent paper of Brockman and Turtle (2003). Their paper starts by viewing the market value of equity, V_E, as a DOC option of the market value of corporate assets V with an exercise price that is equal to the future promised payment (all non-equity liabilities) X with provision at a constant barrier level H. The market value of equity and the future promised payment are obtained from market information and the financial reports of firms, respectively. As the market value of corporate assets is not observable, it is approximated by the proxy ($\tilde{V} = V_E + X$). The asset volatility is measured as the annualized standard deviation of the asset return. By assuming zero rebates for all firms, barriers are calibrated by setting the DOC option formula to the market value of equity. As the DOC price collapses to the SC price when the barrier level is set to zero, their paper claims that the positive barrier hypothesis is testable. Their statistical tests report that implied barriers are significantly positive with over 99% confidence.

Looking at their results carefully, we discover a very interesting phenomenon. In their sample, most (if not all) of the implied barriers H are greater than the future promised payment X. This can be easily deduced from Panel D of Table 2 in their paper, in which the averaged barrier-to-asset ratios are greater than the upper bounds of liability-to-asset ratios. In this section, we show that all of the findings that are mentioned here and in the last paragraph are consequences of using the proxy.

To begin the analysis, we deduce an implication by using the proxy $\tilde{V}$ under the SC framework for security valuation. By putting the equity value as the subject, the definition of the proxy is expressed as

$$V_E = \tilde{V} - X,$$

where the right-hand side is nothing but the intrinsic value of an SC option written on the proxy $\tilde{V}$ with a strike price X. Using the no arbitrage pricing principle, standard textbooks on options, such as that of Hull (2001) and others, give a model-independent result that

$$\tilde{V} - X < \tilde{V} - Xe^{-rT} \leq \mathbf{SC}(\tilde{V}, X),$$

where $\mathbf{SC}(\tilde{V}, X)$ denotes the pricing formula for SC options. As the market value of equity is strictly less than the SC price, this implies that the SC framework will automatically be rejected whenever the proxy is adopted.

In fact, the proxy forces implied barriers to be positive under the DOC approach of Brockman and Turtle (2003). All other things being fixed, the DOC option price is a decreasing function of the barrier level H. Specifically, the DOC price decreases from the SC price to the rebate value R by increasing the barrier level H from 0 to the asset value. With a rebate that is less than the option's intrinsic value, a positive barrier must be chosen to make the DOC price equal to its intrinsic value. This model-independent property of the DOC options implies that a positive barrier will be generated if the proxy is used. We stress that this implication is true for arbitrary sets of input parameters. Some examples of input parameters include industrial sector, the value of the firm's liabilities, asset volatility, option maturity, and rebate level. This explains why the hypothesis tests and robustness tests of Brockman and Turtle (2003) work extra ordinarily well. Firms are presumed to have positive barriers under the their framework.

However, employing the proxy is equivalent to presuming that the default barrier is greater than the future promised payment of liabilities. To highlight this, we denote $\mathbf{DOC}(V, X, H)$ as the current price for a DOC option on V with a strike price of X and a barrier of H. By the no arbitrage pricing principle, we can show that

$$\mathbf{DOC}(V, X, X) > V - X.$$

If this is not the case (that is, if $V - \mathbf{DOC}(V, X, X) - X \geq 0$), then an investor can make an arbitrage profit by selling the asset at V to purchase the DOC option. The remaining cash is put into a bank account. Profits can then be made by taking two different actions that correspond to two possible scenarios.

1. The asset price V does not breach the barrier level X before maturity. On the maturity day (T), the investor will exercise the option to purchase the asset by a value of X so that the investor's short position in the asset will be canceled. An arbitrage profit of

$$[V - \mathbf{DOC}(V, X, X)]\, e^{rT} - X$$

is then made at time T.

2. If the asset value breaches the barrier level X at time $\tau < T$, then the investor will receive a rebate of R. The investor will purchase the asset from the market right away with an amount of X to cancel the short position in the asset. An arbitrage profit of

$$[V - \mathbf{DOC}(V, X, X)]\, e^{r\tau} - X + R$$

is then made at time τ.

As a result, the no arbitrage price of the DOC options should satisfy the preceding inequality. This inequality implies that a DOC option price equals its intrinsic value only when the barrier level H is strictly greater than the strike X. Mathematically, we write

$$\begin{aligned} V - X = \mathbf{DOC}(V, X, H) \quad &\Rightarrow \quad \mathbf{DOC}(V, X, X) > \mathbf{DOC}(V, X, H) \\ &\Rightarrow \quad H > X. \end{aligned} \tag{1}$$

The last line of (1) is true, because the DOC pricing formula is decreasing with the barrier level.

Unfortunately, Brockman and Turtle (2003) use the DOC option pricing formula and the proxy at the same time. This specification is equivalent to setting the DOC option price to be equal to its intrinsic value. According to the last implication (1), all of the "implied barriers" must exceed the book value of corporate liabilities. More importantly, our argument points out a fact that the empirical findings of Brockman and Turtle (2003) are seriously biased toward positive default barriers.

To ensure that we understand the Brockman-Turtle framework correctly, a numerical example is constructed as a verification. We solve H from the equation

$$V - X = \mathbf{DOC}(V, X, H) \tag{2}$$

with various input parameters, where the mathematical formula for the DOC option is presented in (4) of Section 3. We tabulate our results together with that of the robustness tests of Brockman and Turtle (2003) in Table 1 for comparative purposes. To match their scale, we use a market value of corporate assets of 1.0, a future promised payment of 0.45, a risk-free rate of 5%, and a base asset volatility of 25% in our computation. It is important to note that our computation involves no empirical data.

Table 1: Solving (2) vs. Brockman and Turtle (BT, 2003)

Panel A: barrier estimates for various option lives with fixed volatility and zero rebate

	3 Years	5 Years	10 Years	30 Years	100 Years
Solving (2)	0.6543	0.6623	0.6839	0.7208	0.7352
BT	0.6772	0.6802	0.6920	0.7137	0.7224

Panel B: barrier estimates for rebates of 0, 5, 10, 15, and 20%.

	0 %	5 %	10 %	15 %	20 %
Solving (2)	0.6839	0.7067	0.7307	0.7560	0.7825
BT	0.6920	0.7123	0.7334	0.7553	0.7777

Panel C: barrier estimates for volatilities of 80 , 90, 100, 110, and 120% of the base case volatility

	80 %	90 %	100 %	110 %	120 %
Solving (2)	0.7377	0.7091	0.6839	0.6619	0.6425
BT	0.6991	0.6954	0.6920	0.6884	0.6844

The results in Panel A and Panel B of Table 1 show that the barrier levels that are obtained by solving (2) are very close to the averaged normalized barrier levels that are obtained by Brockman and Turtle (2003) in terms of both the order of magnitude and the increasing trend. Panel C of Table 1 reveals that the decreasing trends of the barrier values by the two methods agree with each other. In Panel C, the barriers that are implied by the two methods have slightly different values, because Brockman and Turtle (2003) use the volatility of the proxy, whereas we use a fixed value of 25%. Moreover, all of the "implied barriers" are greater than the liability level of 0.45.

3 The proposed framework

In this section, we propose a statistical framework to estimate default barriers and then verify our approach with a simulation. As the proxy cannot be used, the number of parameters increases from 1, that is the barrier level only, to 4, which includes the default barrier, asset value, asset volatility, and drift of the business at each time point. Therefore, the new framework should be able to manage more parameters and maintain the quality of estimation.

The proposed framework starts by viewing the equity value as a DOC option of the corporate assets. We make the usual assumption that the underlying asset price evolves as a geometric Brownian motion. Specifically, the process for the log-asset-value, $w_t = \ln V_t$, takes the form

$$dw_t = \left(\mu - \sigma^2/2\right) dt + \sigma dZ_t, \tag{3}$$

where V_t is the market value of the firm's assets at time t, σ is the asset value volatility, μ is the drift of the business, and Z_t is a Wiener process. By risk-neutral valuation, equation (3) enables us to derive the closed form solution for the DOC options. In this case, the market value of equity V_E is given as follows:

$$\begin{aligned} V_E &= \mathbf{DOC}(V, X, H) \\ &= VN(a) - Xe^{-rT}N\left(a - \sigma\sqrt{T}\right) \\ &\quad -V(H/V)^{2\eta}N(b) + Xe^{-rT}(H/V)^{2\eta-2}N\left(b - \sigma\sqrt{T}\right) \\ &\quad R(H/V)^{2\eta-1}N(c) + R(V/H)N\left(c - 2\eta\sigma\sqrt{T}\right), \end{aligned} \tag{4}$$

where V is the market value of the firm's assets, X is the future promised payment, H is the barrier level, σ is the asset value volatility, r is the risk-free interest rate, T is the time to maturity of the option, R is the rebate that is paid to the firm's owners if the asset value breaches the barrier, $N(\cdot)$ is the cumulative distribution function for a standard normal random variable, and

$$a = \begin{cases} \frac{\ln(V/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X \geq H, \\ \frac{\ln(V/H)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X < H, \end{cases}$$

$$b = \begin{cases} \frac{\ln(H^2/VX)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X \geq H, \\ \frac{\ln(H/V)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X < H, \end{cases}$$

$$c = \frac{\ln(H/V)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad \eta = \frac{r}{\sigma^2}+\frac{1}{2}. \tag{5}$$

Notice that the SC option framework is incorporated by setting H to zero. This idea comes from Brockman and Turtle (2003).

Given a time series of the market value of equities, say $\{V_E(t_i)|i = 1, 2, \cdots, n\}$, we estimate the drift μ, volatility σ, barrier level H, and a series of market values of corporate assets $\{V(t_i)|i = 1, 2, \cdots, n\}$ by means of a maximum likelihood (ML) estimation. This idea was originated by Duan (1994) in the context of estimating the BSM model. Recently, Ericsson and Reneby (2002) have compared the performances of the ML estimation and the variance-restriction (VR) method by Ronn and Verma (1986). They find that the accuracy of the ML approach is superior to the VR approach. Here, we extend Duan's framework to estimate the default barriers. To our understanding, our paper is the first work to estimate default barriers by a statistical method.

We denote by $f(V_E(t_i)|V_E(t_{i-1}), \theta)$ the probability density function for the equity value at time t_i that is conditional on the equity value at time t_{i-1} and a parameter vector θ. The ML approach estimates the value of θ such that the log-likelihood function

$$L(\theta) = \sum_{i=2}^{n} \ln f\left(V_E^i|V_E^{i-1}, \theta\right), \quad V_E^i \equiv V_E(t_i)$$

is maximized. For our case, $\theta = (\mu, \sigma, H)$. If the density function can be expressed in closed form, then the maximization problem becomes tractable.

Fortunately, the conditional density function $f(\cdot|\cdot)$ for the equity value can be derived from the DOC option pricing formula of (4). We denote $g(w_i|w_{i-1}, \theta)$ as the density function of w_i that is conditional on the values of w_{i-1} and θ, where w_i is the log-asset-value at time t_i. With the help of

(4), a standard change of variable technique is applied to obtain

$$f(V_E^i|V_E^{i-1},\theta) = \left[g(w_i|w_{i-1},\theta) \times \left| \frac{\partial V_E}{\partial w} \right|^{-1} \right]_{w_i=w\left(V_E^i,t_i;\underline{\theta}\right)}, \tag{6}$$

where $\underline{\theta} \subset \theta$ is the subset of the parameter vector, which is necessary for pricing the equity values. In fact, w_i is obtained inversely from (4) with the values of V_E^i and $\underline{\theta} = (\sigma, H)$ given. This is because the DOC pricing formula does not involve the drift μ. The partial derivative that appeares in (6) can be implemented through the delta of the DOC options

$$\left.\frac{\partial V_E}{\partial w}\right|_{w=w_i} = V_i \left.\frac{\partial V_E}{\partial V}\right|_{V=V_i} = V_i \Delta(V_i, \theta).$$

The conditional density function, $g(\cdot|\cdot)$, for the random variable w_i should reflect an absorbing boundary condition on $w_i = \ln H$ such that the asset values will not fall under the barrier between any two successive time points. In other words, we are considering survival firms. This density function is available in the literature, see Rubinstein and Reiner (1991). Specifically,

$$g(w_i|w_{i-1};\theta) \;=\; \varphi(w_i - w_{i-1}) - e^{2\eta(h-w_{i-1})}\varphi(w_i + w_{i-1} - 2h), \tag{7}$$

where

$$\begin{aligned} \delta t_i &= t_i - t_{i-1}, \quad h = \ln(H), \\ \varphi(x) &= \frac{1}{\sigma\sqrt{2\pi\delta t_i}} \exp\left\{ -\frac{[x - (\mu - \sigma^2/2)\delta t_i]^2}{2\sigma^2 \delta t_i} \right\}, \end{aligned} \tag{8}$$

and η is defined in (5). It is important to note that the function $g(\cdot|\cdot)$ takes the form (7) if the underlying asset value is larger than the barrier, otherwise, its value is set to zero. Our approach can be further improved by taking into account the survivorship consideration (see Duan et al. (2003)). However, the improvement is mainly related to the drift, and does not affect the other parameters. As our basic concern is the default barrier, we keep this present setting unchanged to avoid further complication.

Let us summarize the whole estimation procedure. After specifying the log-likelihood function as

$$L(\theta) \;=\; \sum_{i=2}^{n} \left[\ln g(w_i|w_{i-1}) - \ln\left[V_i \Delta(V_i;\underline{\theta})\right]\right]_{w_i=w(V_E^i,t_i;\underline{\theta})}, \tag{9}$$

the drift μ, the volatility σ, and the barrier H are estimated by maximizing equation (9). Finally, a time series of asset values is obtained from the inverse of the equity pricing function

$$\widehat{w_i} = DOC^{-1}\left(V_E^i, t_i; \hat{\sigma}, \widehat{H}\right).$$

3.1 Simulation checks

A Monte Carlo simulation is designed to check the reliability of our ML estimation. Through simulation studies, we are able to obtain some idea of the performance, in terms of strengths and weaknesses, of our framework. We also compare the proposed ML estimation with the approach of Brockman and Turtle (2003). As we choose the parameter values, we can check the accuracy of the two approaches.

The simulation generates 1,000 realizations of asset values in accordance with the dynamics of (3). Each realization comprises 2,600 equally time-spaced asset values, that replicate 1-year intraday observations. In other words, we assume that there are 10 observations per day and 260 trading days per year. We take the following parameter values: $r = 0.05, T = 10, \mu = 0.1, \sigma = 0.3, R = 0, X = 1.0$, and $V(0) = 1.5$. For each generated realization, three sets of equity values are produced via the DOC option pricing formula for $H = 0.8, 1.0$, and 1.2. To obtain positive equity values at each time point, we regenerate the realization that contains an asset value that is less than the barrier. Ultimately, we obtain 1,000 time series of equities for each value of H.

Because we only obtain daily closing prices in our empirical study, we sample 260 equally time-spaced points from each generated equity realization. This allows us to capture the effect of sampling from discrete time points. By viewing these 260 $\times$ 1,000 equity values as real data that are observed in the market, we perform our ML estimation and the Brockman and Turtle (2003) estimation. The results are summarized in Table 2.

Panel A of Table 2 reports the barrier estimates. It shows that the accuracy of the H estimates is high regardless of whether the true value of the default barrier is less than, equal to, or greater than the liability level of 1.0. The averaged barrier estimates are 0.7857, 1.0061, and 1.2036 for the true values of 0.8, 1 and 1.2, respectively. All of the averaged values are close to the corresponding median values. The standard deviation decreases with the true barrier level, and therefore the performance of the barrier estimation improves for higher values of default barriers. Brockman and Turtle (2003) claim that default barriers are significantly positive. If this is really the case, then our approach captures default barriers even better. When we look at the results that are obtained from the Brockman and Turtle (2003) approach, we can see that the implied barriers are significantly higher than the corresponding true values. For a true barrier of 0.8, the proxy approach gives an estimate of 1.4577, which has a percentage error of over 80%. The upward bias that is inherent in the proxy approach is an obvious conclusion that can be drawn from these figures.

Table 2: Performamce of the estimations

	ML estimation			BT approach		
Panel A: barrier estimates (true value = barrier level)						
barrier level:	0.8	1.0	1.2	0.8	1.0	1.2
mean:	0.7857	1.0061	1.2036	1.4557	1.3402	1.2114
std:	0.3551	0.2610	0.2063	0.0836	0.0918	0.0893
median	0.8247	1.0245	1.2236	1.4439	1.3274	1.1972
Panel B: volatility estimates (true value = 0.3)						
barrier level:	0.8	1.0	1.2	0.8	1.0	1.2
mean:	0.2949	0.3003	0.3013	0.2586	0.3113	0.4051
std:	0.0560	0.0707	0.0751	0.0129	0.0199	0.0428
median	0.2937	0.2943	0.2936	0.2582	0.3102	0.4034
Panel C: drift estimate (true value = 0.1)						
barrier level:	0.8	1.0	1.2	0.8	1.0	1.2
mean:	0.3838	0.3810	0.3753	0.2242	0.2691	0.3538
std:	0.5260	0.5325	0.5328	0.1962	0.2211	0.2617
median	0.1605	0.1502	0.1419	0.2056	0.2551	0.3499
Panel D: Percentage error of asset values (true value = 0)						
barrier level:	0.8	1.0	1.2	0.8	1.0	1.2
mean:	0.04729	0.05611	0.05314	0.2220	0.1465	0.0473
std:	0.03985	0.04999	0.04888	0.0202	0.0074	0.0163
median	0.03582	0.04429	0.04305	0.2232	0.1467	0.0456

Panel B shows the performance of the volatility estimates. The maximum likelihood estimators, in terms of both the means and the medians, are very close to the true value of 0.3. Moreover, the standard derivation for each volatility estimate is smaller than 0.08. The performance is good for all values of default barriers. Although the bias in the volatility estimates of Brockman and Turtle (2003) is less severe compared to the case of the default barriers, the error is large enough to generate misleading messages in practice. When the Brockman and Turtle (2003) approach roughly captures the barrier at $H = 1.2$, it overestimates the volatility by over 33%.

Panel C shows that the drifts are overestimated in both approaches. This is to be expected, as only survival firms are considered in this simulation. Although this bias can be reduced, [1] the drift is often irrelevant in application. For pricing credit derivatives or corporate bonds with structural models, valuations are carried out in a risk-neutral world in which the drift has no role in the pricing formulas. Moreover, this paper concentrates on the default barrier estimates, and thus the errors in the estimation of the drift does not affect our general discussion.

In Panel D, we calculate the percentage error of estimated asset values relative to the true values using the following formula:

$$\begin{array}{c}\text{Percentage error}\\ \text{of firm asset values}\\ \text{in one simulation}\end{array} = \frac{1}{260}\sum_{i=1}^{260}\left|\frac{V^i_{true} - \widehat{V^i}}{V^i_{true}}\right|, \tag{10}$$

where $\widehat{V^i}$ is the estimated firm value. The averaged percentage errors are less than 6% for all cases using the ML approach, and the medians are even smaller. Altogether, our simulation gives evidence that the proposed ML framework renders a precise estimation of default barriers, firm asset value, and asset volatility. With the Brockman and Turtle (2003) framework, the firm values are wrongly estimated by 4.73% - 22.2% on average across different cases. The estimation error increases when the barrier level decreases.

[1] Duan et al. (2004) have modified our framework to remove this bias. However, the estimation quality of other parameters remains the same. The aim of their paper is to argue that ML estimation is a more general approach than the EM algorithm that is employed by the KMV. Our framework serves as a good example to strengthen their arguments.

4 Estimation with empirical data

4.1 Description of the data

This section presents an empirical investigation over a large cross-section of industrial firms. We collected data from Compustat and Datastream. Attention is paid to industrial firms with SIC codes that are between 2,000 and 5,999. The sample covers a ten-year period of *daily observations* from 1993 to 2002. The whole data set consists of 13,317 firm-years, which provides abundant data in various industrial sectors to perform the ML estimation.

Table 3 presents the basic statistics of our data. It can be seen that the debt-to-equity ratio varies greatly across the different sectors, and ranges from around 0.001 to over 5000. This implies that the sample for our empirical analysis takes a large number of firms with various financial structures into account. When all observations are pooled together, it is found that the averaged face value of debts of a firm is about 2.6 times to its market equity value. The annualized risk-free rate ranges from the lowest of 1.32% to the highest of 5.98% in the period. The mean value is 4.3%.

<< *Insert Table* 3 *about here* >>

To estimate the default barrier H, the market value of assets (V), and the firm asset volatility (σ), the ML approach requires the market value of equity (V_E), the future promised payment (X), the risk-free interest rate (r) and the time to maturity of the option (T) as inputs. The equity values are directly obtained from Compustat and Datastream. The data of X are measured as the book value of assets minus the book value of equity, both of which were downloaded from Compustat. We take the rate of return of one-year US Treasury bills as the risk-free interest rate. The option's maturity is assumed to be 10 years, which has been used in many empirical studies, such as that of Brockman and Turtle (2003). This allows us to make a fair comparison.

We implement the estimation with the following technical details. The time series of V_E together with other values are substituted into the log-likelihood function of (9). This is then maximized through the numerical scheme of Nelder-Mead (1965) that is a built-in function of the software MATLAB. A convergence analysis for the scheme is reported in Lagarias et al. (1998).

4.2 Empirical results

4.2.1 Barrier vs. debt

In Section 2, we show theoretically that the proxy approach of Brockman and Turtle (2003) lead to barrier level (H) above the value of corporate liabilities (X), and that their result is thus unable to reflect reality. Our empirical study supports this claim, as we find that default barriers tend to be less than the value of X. Table 4 reports the statistics of the barrier-to-debt ratio (H/X), which allows us to compare the barriers with the value of liabilities directly. We use percentiles to report the barrier-to-debt ratio, because the average values can generate a distorted picture. In our sample, some typical firms have a small value (close to zero) in their liabilities, see Table 3. In such a situation, the computational error in the barrier estimates greatly affects the accuracy of measuring the average values. If the debt level is close to zero, then the estimated barrier is small, but it is possible for it to be larger than the debt level due to the computational error. The resulting barrier-to-debt ratio can be unreasonably large. When we measure the average values, the number dominates the outcome by pulling up the mean value significantly.

<< *Insert Table* 4 *almost here* >>

Panel A of Table 4 shows the statistics across the whole sample. We can see that over 25% of the sample has zero default barriers. This result is significantly different from the finding of Brockman and Turtle (2003) of a 99% confidence that default barriers are positive and significantly different from zero. The median value is 0.738, which indicates that the median firm has a default barrier at around 74% of its liabilities. Interestingly, for a firm with 50% long-term debt and 50% short-term debt, the default point, or the specified default barrier, of KMV becomes 75% of the total debt which is close to the median barrier-to-debt ratio that is given by our sample. In fact, 55.7% of the whole sample has a ratio of less than 1. This suggests that the default barrier of most firms is lower than their level of liabilities. However, Brockman and Turtle (2003) implicitly show that default barriers are higher than the total debt. We disaggregate the sample by years and by industrial sectors to find other useful information.

Panel B presents the results by different years. The medians of the barrier-to-debt ratios are consistently less than 1, except for the years 1998 and 2002. It is interesting to recall that there are shocks on the major financial indices in both years. We provide three possible explanations for this observation. During the period of financial distress, investors overreact to all kinds of risks, including the default risk, and thus price equities with a higher level of default barriers. As our framework takes into account the market values of equities, this effect is successfully captured. The second possible reason is concerned with model risk. Jumps on equity values arrive more frequently in years of financial distress to pull down stock prices. Our framework only considers a continuous time model without jumps, and thus an unreasonable decrease in equity value leads to the overestimation of the volatilities. A high volatility generates a high equity value with the DOC option model. However, the equity value is a known quantity, and thus a high barrier is implied to balance off the effect. The third reason is a combination of the previous two reasons, and therefore, future research should focus on the impact of jumps on the default barrier estimation.

We look at the default barrier estimates across industrial sectors in Panel C. Except for the chemicals sector and the miscellaneous manufacturing sector, the medians of the sectors are all less than 1. When we refer to Table 3, the median of the debt-to-equity ratios of these two sectors are the smallest two among all the industrial sectors. This observation suggests that the lower the debt-to-equity ratio, the higher the barrier-to-debt ratio. A possible explanation is that firms are subject to bankruptcy costs, which contribute to the value of default barrier. When the debt value is typically low, we expect to observe a barrier that is above the debt for a median firm in a sector. However, the impact of bankruptcy costs on the default barrier is not answered in this paper, and we leave it to future research to determine this impact.

4.2.2 Market value of corporate assets

In the simulation study, we show that the firm asset value and volatility are estimated with a high degree of accuracy with our ML approach. This enables us to examine the quality of the firm asset value estimation with the proxy. To make this comparison, we use the following measurement:

$$\begin{array}{c}\text{Percentage difference of}\\ \text{firm asset values}\\ \text{for a firm year}\end{array} = \frac{1}{N_y}\sum_{i=1}^{N_y}\frac{V^i_{proxy} - V^i_{ML}}{V^i_{ML}}, \tag{11}$$

where N_y is the number of trading days in a particular year, and V^i_{proxy} and V^i_{ML} are, respectively, the firm value that is estimated by the proxy and the ML estimation at time t_i.

Table 5 shows the mean and standard deviation of the percentage error. The mean value is 0.667, which shows that the proxy approach overestimates the firm asset value by 66.7% on average. However, the large standard deviation indicates that it is possible to have a case in which the proxy approach underestimates the firm value. If we regard the ML estimation as a benchmark, as it is asymptotically unbiased, then the proxy approach is generally very inaccurate. However, we caution that the proxy should be avoided in an empirical study if the default barrier is considered in an empirical study.

Table 5: Percentage error in estimating firm values

	mean	standard deviation
Error of firm values	0.662	17.598

4.2.3 Volatility

We also examine the bias in asset volatility that is generated through the proxy approach. In Table 6, the percentage difference between the firm asset volatility as estimated by the proxy approach of Brockman and Turtle (2003) and that which is obtained by our ML method is reported. We denote by σ_{proxy} the annualized standard derivation of the proxy asset values. The comparison is based on the following measurement:

$$\begin{array}{c}\text{Percentage difference of}\\ \text{firm asset volatility}\\ \text{for a firm}\end{array} = \frac{\sigma_{proxy} - \sigma_{ML}}{\sigma_{ML}}. \tag{12}$$

In Table 6, it can be seen that the volatilities that are obtained using the approach of Brockman and Turtle (2003) deviate substantially from those that are derived from the ML estimation. The former approach overestimates the volatility by 74.8% on average.

Table 6: Percentage error in estimating volatility

	mean	standard deviation
Error of volatilities	0.748	14.78

4.2.4 A final word on the empirical result

Up to this point, we have cast doubt on the ability of the Brockman and Turtle (2003) framework to explain the effect of the default barrier in corporate claims. However, this does not mean that our approach is perfect. There are many ways to improve the approach, such as taking into account the bankruptcy codes, bond values, and other factors. Nevertheless, it is crucial for us to choose an estimator or proxy that does not create any internal conflicts or obvious bias. As empirical analysis is a kind of statistical work, we suggest using statistical estimation whenever possible, because proxy errors can sometimes cause serious mistakes.

5 Conclusion

This paper shows theoretically that using the sum of the market value of equity and the book value of corporate liabilities as a proxy for the market value of corporate assets leads to an upward bias in the estimation of the default barrier. To capture default barriers from market information, a maximum likelihood estimation is proposed to estimate the barrier, the market value of corporate assets, and the asset value volatility. We test the performance of the proposed approach with a simulation, and find that it has a good estimation quality. The proposed framework is applied to an empirical study, and we show that most of the firms in the study have a positive default barrier that is less than the book value of corporate liabilities. The corporate asset values are also overstated by the proxy approach, and the asset volatility that is implied by the proxy firm values is also unrealistic. Using this study, we spell out the risk of using proxies in parameter estimation and appeal to researchers to be aware of this risk.

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Table 3
Descriptive statistics for the sample

Panel A: Debt to equity ratio

Industrial sector	Number of firm years	Minimum	Median	Maximum	Mean	Standard Derivation
1. Food and beverages (20)	382	0.0336	0.4306	29.3338	1.1030	2.8101
2. Miscellaneous (21,24,25,27,30,31,46,48)	1557	0.0066	0.6865	644.7465	2.3708	17.6009
3. Textiles and apparel (22,23)	231	0.0200	0.7526	1067.2545	10.1094	80.4994
4. Paper products(26)	320	0.0091	1.0545	17.0665	1.6308	1.9783
5. Chemicals (28)	1948	0.0016	0.1689	252.5202	1.5408	12.1809
6. Petroleum (29)	149	0.0515	0.6893	9.7840	1.0849	1.1989
7. Stone, clay, and glass (32)	139	0.0088	0.7941	143.3097	3.0990	13.9116
8. Primary metals (33)	332	0.0274	1.0348	196.6046	3.4254	12.1530
9. Fabricated metals (34)	358	0.0604	0.6718	97.4935	1.4833	5.4988
10. Machinery (35)	1260	0.0030	0.3479	18.8429	0.6998	1.1738
11. Appliances, electrical equipment (36)	1449	0.0025	0.2786	5554.7237	9.4925	177.5735
12. Transportation equipment (37)	422	0.0117	0.8551	173.2681	2.6338	12.1227
13. Miscellaneous manufacturing (38,39)	1598	0.0015	0.2049	95.2583	0.6857	3.0561
14. Railroads (40)	96	0.1469	1.2535	6.4642	1.5994	1.1533
15. Other transportation (41,42,44,45,47)	542	0.0230	1.3396	59.7711	2.9580	5.9337
16. Utilities (49)	971	0.0199	1.4568	114.6390	2.2389	5.8396
17. Other retail trade (50-52, 54-59)	1536	0.0031	0.6544	49.9581	1.5456	3.0665
18. Department stores(53)	27	0.0616	1.6225	4.8043	1.5707	1.4832
Pooled result	13317	0.0015	0.5151	5554.7237	2.6600	60.1983

Panel B: Risk free rate

Mean	Standard Derivation	Minimum	Median	Maximum
0.0428	0.0148	0.0132	0.0486	0.0598

Table 4
Barrier-to-debt ratio (H/X)

	Number of firm years	percentile 5%	25%	50%	75%	95%	proportion of $H < X$
Panel A: Pool sample results							
Pooled	14333	0.000	0.000	0.738	2.475	18.51	0.557
Panel B: Barrier-to-debt ratio by year							
1993	938	0.000	0.000	0.514	2.246	14.82	0.587
1994	1048	0.000	0.000	0.844	2.637	17.29	0.532
1995	1115	0.000	0.000	0.125	1.601	8.29	0.650
1996	1277	0.000	0.000	0.702	2.493	25.38	0.556
1997	1403	0.000	0.000	0.185	2.167	18.48	0.617
1998	1524	0.000	0.277	1.432	3.919	21.67	0.389
1999	1630	0.000	0.000	0.351	1.472	10.97	0.671
2000	1821	0.000	0.000	0.594	2.274	28.00	0.614
2001	1813	0.000	0.000	0.719	2.388	19.29	0.558
2002	1764	0.000	0.029	1.352	3.600	18.43	0.431

Table 4 (continued)

	Number of firm years	percentile 5%	25%	50%	75%	95%	proportion of $H < X$
Panel C: Barrier-to-debt by sectors							
1. Food and beverages (20)	392	0.000	0.000	0.545	2.727	12.86	0.561
2. Miscellaneous (21,24,25,27,30,31,46,48)	1642	0.000	0.000	0.608	1.847	6.08	0.590
3. Textiles and apparel (22,23)	250	0.000	0.000	0.521	1.679	8.09	0.656
4. Paper products(26)	305	0.000	0.000	0.653	1.598	4.25	0.590
5. Chemicals (28)	2271	0.000	0.000	1.477	7.318	48.11	0.445
6. Petroleum (29)	168	0.000	0.031	0.695	1.500	2.97	0.601
7. Stone, clay, and glass (32)	115	0.000	0.000	0.669	2.076	13.02	0.583
8. Primary metals (33)	380	0.000	0.000	0.630	1.395	4.44	0.634
9. Fabricated metals (34)	394	0.000	0.000	0.801	2.033	8.17	0.553
10. Machinery (35)	1376	0.000	0.000	0.941	2.950	15.60	0.515
11. Appliances, electrical equipment (36)	1618	0.000	0.000	0.891	3.147	23.61	0.518
12. Transportation equipment (37)	430	0.000	0.000	0.685	1.941	9.46	0.586
13. Miscellaneous manufacturing (38,39)	1682	0.000	0.001	1.247	5.661	32.75	0.465
14. Railroads (40)	103	0.000	0.000	0.295	1.060	3.07	0.728
15. Other transportation (41,42,44,45,47)	525	0.000	0.000	0.581	1.290	3.67	0.661
16. Utilities (49)	1035	0.000	0.000	0.020	0.925	1.83	0.781
17. Other retail trade (50-52, 54-59)	1623	0.000	0.000	0.608	1.816	9.39	0.614
18. Department stores(53)	24	0.000	0.000	1.000	1.514	19.70	0.500