-Matrices and Linear Equations

-matrix form of the system of linear equations: \( Ax = B \)

-the associated homogeneous system: \( Ax = 0 \)

-Theorem:

Suppose \( x_1 \) is a particular solution to the system \( Ax = B \) of linear equations. Then every solution \( x_2 \) to \( Ax = B \) has the form

\[ x_2 = x_0 + x_1 \]

for some solution \( x_0 \) of the associated homogeneous system \( Ax = 0 \).

-Theorem:

Consider the homogeneous system \( Ax = 0 \) in \( n \) variables where \( A \) has rank \( r \). Then:

1. The system has exactly \( n - r \) basic solutions.

2. Every solution is a linear combination of the basic solutions.

Example:

\[
\begin{align*}
x_1 + 2x_2 - 3x_3 + 2x_4 &= 2 \\
2x_1 + 5x_2 - 8x_3 + 6x_4 &= 5 \\
3x_1 + 4x_2 - 5x_3 + 2x_4 &= 4
\end{align*}
\]
-Inverse
Let $A$ be a square matrix. $A^{-1}$ is the inverse of $A$ if and only if
$$AA^{-1} = I \quad \text{and} \quad A^{-1}A = I$$

-Theorem:
Suppose a system of $n$ equations in $n$ variables is written in matrix form as
$$Ax = B.$$ 
If the $n \times n$ coefficient matrix $A$ is invertible, the system has the unique solution
$$x = A^{-1}B$$

-Matrix Inversion Algorithm

$$[A|I] \rightarrow [I|A^{-1}]$$

-Properties of Inverses
-refer to your notes or book

Example:
In each case either prove the assertion or give an example showing that it is false.

(a) If $A \neq 0$ is a square matrix, then $A$ is invertible.
(b) If $A$ and $B$ are both invertible, then $A + B$ is invertible.
(c) If $A$ and $B$ are both invertible, then $(A^{-1}B)^T$ is invertible.
(d) If $A^4 = 3I$, then $A$ is invertible.
(e) If $A^2 = A$ and $A \neq 0$, then $A$ is invertible.