Vectors and Lines

- vector

- length

- equal vectors (same length and same direction)

- vector addition
  
  Basic properties
  
  (a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ for all vectors $\mathbf{u}$ and $\mathbf{v}$.
  (b) $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.
  (c) $\mathbf{v} + \mathbf{0} = \mathbf{v}$ for all vectors $\mathbf{v}$.
  (d) $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$ for all vectors $\mathbf{v}$.

- $\vec{AB}$ (vector from $A$ to $B$)

- scalar multiplication

- Unit vector
Example
Show that the midpoints of the four sides of any quadrilateral are the vertices of a parallelogram.

**Theorem:**
Two nonzero vectors $\mathbf{v}$ and $\mathbf{w}$ are parallel if and only if one is a scalar multiple of the others.
Coordinates and Lines

- position vector

- coordinate vectors

- direction vector for a line
Vector Equation of a Line

The line parallel to $d \neq 0$ through the point with position vector $p_0$ is given by

$$p = p_0 + td$$

for some scalar $t$.

In other words, the point with position vector $p$ is on this line if and only if a real number $t$ exists such that $p = p_0 + td$.

Parametric Equations of a Line

The line through $P_0(x_0, y_0, z_0)$ with direction vector $d = (a, b, c) \neq 0$ is given by

$$x = x_0 + ta$$
$$y = y_0 + tb$$
$$z = z_0 + tc$$

where $t$ is any scalar. In other words, the point $P(x, y, z)$ is on this line if and only if a real number $t$ exists such that $x = x_0 + ta, y = y_0 + tb$, and $z = z_0 + tc$. 
Example:

Let $A$, $B$, and $C$ denote the three vertices of a triangle. If $E$ is the midpoint of side $BC$, show that

$$\vec{AE} = \frac{1}{2}(\vec{AB} + \vec{AC}).$$

Example:

Determine whether $\mathbf{u}$ and $\mathbf{v}$ are parallel if $\mathbf{u} = (-3, -6, 3)$; $\mathbf{v} = (5, 10, -5)$. 
Example:
Let $\mathbf{u}$ and $\mathbf{v}$ be the position vectors of points $P$ and $Q$, respectively, and let $R$ be the point whose position vector is $\mathbf{u} + \mathbf{v}$. Express the following in terms of $\mathbf{u}$ and $\mathbf{v}$.

(a) $\vec{QP}$
(b) $\vec{QR}$

Example
Let $P(1, -1, 3)$ and $Q(3, 1, 0)$ be two points. Find $PQ$ in component form.

Example
Let $\mathbf{u} = (1, 1, 2)$, $\mathbf{v} = (0, 1, 2)$, and $\mathbf{w} = (1, 0, -1)$. Find numbers $a, b,$ and $c$ such that $\mathbf{x} = a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$ where $\mathbf{x} = (2, -1, 6)$. 
Example

Find two points trisecting the segment between $P(2, 3, 5)$ and $Q(8, -6, 2)$. 
Example

Find the vector and parametric equations of the following line: The line parallel to \((2, -1, 0)\) and passing through \(P(1, -1, 3)\).
Example

Find the point of intersection (if any) of the following pair of lines

\[ x = 3 + t \]
\[ y = 1 - 2t \]
\[ z = 3 + 3t \]

and

\[ x = 4 + 2s \]
\[ y = 6 + 3s \]
\[ z = 1 + s \]
Example

Show that the line through $P_0(x_0, y_0)$ with slop $m$ has direction vector $d = (1, m)$ and equation $y - y_0 = m(x - x_0)$. 