RMS4001 Assignment 5

Due date: April 8, 2004 (before 5pm). You may form a group of no more than 3 students to complete this assignment.

1. Outline the algorithm that generates a random variable having a density function

   \[ f(x) = xe^{-x}, \quad 0 \leq x < \infty, \]

   using the following three methods:

   (a) the inverse transform method;

   (b) the rejection method with the Cauchy density as a proposal;

   (c) the approach for generating the Gamma density.

   Which method is the most efficient one?

2. Valuing a half-year European call option whose payoff is \( \max(S - K, 0) \). Suppose the current asset price of $40, strike price of $40, interest rate of 5% and volatility of 25%.

   (a) Construct a Monte Carlo simulation to value the European call option based on \( X_1, X_2, \cdots, X_{1000} \sim N(0,1) \) iid. Write your program as call.ssc.

   (b) Modify your program by first generating \( X_1, \cdots, X_{500} \sim N(0,1) \) iid and then setting \( X_{i+500} = -X_i, \ i = 1, 2, \cdots, 500 \). Save your modified version as mcall.ssc.

   (c) Repeat (a) and (b) for 100 times. Which method gives a better estimate to the option price? You may use the Black-Scholes formula to check your result.

3. Suppose we want to generate random variables of the generalized error distribution (GED), defined in the lecture notes, by rejection method.

   (a) Can we use normal distribution as a proposal distribution? Why?

   (b) Program the pdf, \( f(x) \), of the positive GED as pged.ssc and plot the density function for \( \nu = 1.2, 1.6 \) and 2 in a single graph.

   (c) Denote \( g(x) \) as the pdf of exponential(1) distribution and \( R(x) = f(x)/g(x) \). Program \( R(x) \) as ratio.ssc and plot \( R(x) \). Estimate the maximum value of \( R(x) \) from the graph if \( \nu = 1.5 \).
(d) Construct a rejection method to generate $X \sim GED(1.5)$ with exponential(1) as the proposal distribution. Write down your algorithm.

4. The asset return for Hirisk Corporation is modeled as

$$R = 0.1 + 0.4\epsilon.$$ 

(a) Estimate the 95% and 99% VaR(zero) if $\epsilon \sim \mathcal{N}(0, 1)$. What is the confidence interval for your estimate? (Use either closed form or simulation)

(b) Estimate the 95% and 99% VaR(zero) if $\epsilon$ follows a t-distribution with 2 degrees of freedom. What is the confidence interval for your estimate? (Use simulation)

(c) Estimate the 95% and 99% VaR(zero) if $\epsilon \sim GED(1.5)$. What is the confidence interval for your estimate? (Use simulation)

(d) Estimate the 95% and 99% VaR(zero) if $\epsilon \sim \mathcal{N}(0, 1 + \delta)$ where $\delta = 1$ with probability 0.2 and 0 with probability 0.8. What is the confidence interval for your estimate? (Use simulation)

(e) Suppose the market return is 6%. What are the RAROCs obtained from the above three models?

5. (Risk assessment of toxic waste) A small community gets its water from wells that tap into an old, large aquifer. Recently, an environmental impact study found toxic contamination in the groundwater due to improperly disposed chemicals from a nearby manufacturing plant. Estimates contaminant's concentration (CC) in the water is measured in micrograms per liter. The cancer potency factor (CPF) for each chemical is uncertain. The CPF is the magnitude of the impact the chemical is. The population risk assessment must account for the variability of body weights and volume of water consumed by the individuals in the community per day. All these factors lead to the following equation for population risk:

$$\text{Population risk} = \frac{CPF \times CC \times W}{BW},$$

where $W = \text{water consumed per day} \sim \mathcal{N}(2, 1)$ that is truncated below zero, $BW = \text{body weight} \sim \mathcal{N}(70, 100)$. We assume that CC follows a triangular distribution with vertices at $x = 80, 110$ and 120 and $\ln(\text{CPF}) \sim \mathcal{N}(0.03, 0.004)$. Suppose all the relevant factor are independent to each other. Simulate the population risk and draw the forecast distribution for the population risk.