GEE 249N
Suggested Solution for Assignment 2

1. Speculation is different from pure gambling. For gambling, the expected payoff is negative. Gambler is risk seeker. However, the expected payoff of speculation in futures markets is zero as it is a zero sum game. Instead of harming the market, speculation has several positive functions towards the efficiency of the market. First, it provides liquidity to the market. As more speculators participate in the market, the market depth will increase. With the increased market depth, liquidity increase and implicit transaction cost (trading spread) decreases. Second, Speculators will also seek for opportunities to gain profit. If the price is significant deviate from the fair value, they will use counter strategy to make the market price closer to the fair price.

2. \( 90000/30000 = 3 \)
The framer can short 3 hogs futures contracts for hedging. Three months later, the framers then buy back three futures to close the position.

The pro of hedging is the shift of the risk to the long futures holders. It can protect the farmer’s income from the price drop of the hogs. However, it is not a free lunch. Farmers undergo hedging will forgone the plausibility of price increase of the hogs.

3. The future price of a stick index may be greater than, equal to or less than the expected future value of the index. The cost of carry will determine it’s relative value of the expected future value of the index. Positive (negative) cost of carry will leads the future price less (greater) than the expected future value.

4. Borrowing $49.5 and buy the put and stock initially.

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>At Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Stock Price&lt;=50</td>
</tr>
<tr>
<td>Borrowing</td>
<td>49.5</td>
<td>-49.7475</td>
</tr>
<tr>
<td>Buy Put</td>
<td>-2.5</td>
<td>Exercise the put and deliver the stock 50</td>
</tr>
<tr>
<td>Buy Stock</td>
<td>-47</td>
<td></td>
</tr>
<tr>
<td>Net Cash Flow</td>
<td>0</td>
<td>+0.2525</td>
</tr>
</tbody>
</table>

5. Lower Bound= \( \max( KB-S, 0) \)
\[
= \max(75/[1+10%/6]- 80, 0) \\
= 0
\]

6. Risk-neutral probability of up = \[
\frac{\left(1+ \frac{0.08}{12}\right)^{-0.9}}{1.1-0.9} \\
= 0.5333
\]

Value of Call = \( (0.5333\times3 +0.4667\times0)/(1+0.08/12) \)
\[
= 1.6 / 1.006667
\]
7. Long options with strike $K_1$, $K_3$ and short two options with strike price $K_2$.
At maturity

<table>
<thead>
<tr>
<th>$S_T \leq K_1$</th>
<th>$K_1 &lt; S_T \leq K_2$</th>
<th>$K_2 &lt; S_T \leq K_3$</th>
<th>$K_3 &lt; S_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of option with strike $K_1$</td>
<td>0</td>
<td>$S_T - K_1$</td>
<td>$S_T - K_1$</td>
</tr>
<tr>
<td>Value of option with strike $K_2$</td>
<td>0</td>
<td>0</td>
<td>$-2(S_T - K_2)$</td>
</tr>
<tr>
<td>Value of two options with strike $K_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Net value of the portfolio</td>
<td>0</td>
<td>$S_T - K_1 &gt; 0$</td>
<td>$2K_2 - K_1 - S_T$</td>
</tr>
</tbody>
</table>

$= (K_2 - K_1) - (S_T - K_2)$

$> (K_2 - K_1) - (K_3 - K_2)$

Since the payoff of the above portfolio is either positive or zero and the no arbitrage pricing principal, the value of this portfolio must greater than or equal to zero.

$C_1 + C_3 - 2C_2 \geq 0$

$C_1 + C_3 \geq 2C_2$

$\frac{C_1 + C_3}{2} \geq C_2$

8. Risk-neutral probability of up $= \frac{1.04 - 0.9}{1.1 - 0.9} = 0.7$

Value of the call $= \frac{0.7 \times 0.7 \times (121 - 100) + 2 \times 0.7 \times 0.3 \times 0 + 0.3 \times 0.3 \times 0}{(1.04)^2}$

$= 10.29 / 1.0816$

$= 9.514$

Value of the put $= \frac{0.7 \times 0.7 \times 0 + 2 \times 0.7 \times 0.3 \times (100 - 99) + 0.3 \times 0.3 \times (100 - 81)}{(1.04)^2}$

$= 2.13 / 1.0816$

$= 1.969$

Put Call Parity

$C + KB = P + S$

L.H.S. = 9.514 + 100 / 1.0816

= 101.969

R.H.S. = 1.969 + 100

= 101.969

L.H.S. = R.H.S.