[Definition] (Independence)

(1) Two events $A, B \in \mathcal{F}$ are statistically independent if

$$P(A \cap B) = P(A)P(B).$$

(2) Two sigma fields $\mathcal{F}_1$ and $\mathcal{F}_2$ are independent if for any two events $A_1 \in \mathcal{F}_1, A_2 \in \mathcal{F}_2$, $A_1, A_2$ are independent.

(3) Two random variables $X_1$ and $X_2$ are independent if

$$P(X_1 \in B_1, X_2 \in B_2) = P(X_1 \in B_1)P(X_2 \in B_2),$$

for any $B_1, B_2 \in \mathcal{B}$.

[Exercise 3.12]

How to define the independence between an event and a random variable? How to define the independence between an event and a $\sigma$-field? How to define the independence between a random variable and a $\sigma$-field?
[Exercise 3.13]

Show that two random variables $X$ and $Y$ are independent if and only if the $\sigma$-fields $\sigma(X)$ and $\sigma(Y)$ are independent.

[Exercise 3.14]

Let $X \sim N(0, 1)$ and $Y = X^2$. Show that $X$ and $Y$ are uncorrelated but dependent.