[Example: Option Pricing in FX Market]

Suppose that in the US dollar markets the current Sterling exchange rate is 1.5. Consider a European call option that offers the holder the right to buy 100 pounds for 150 US dollars at time $T$. The riskless borrowing borrowing rate in the UK is $u$ and that in the US is $r$. Assuming a single period binary model in which the exchange rate at the expiry time is either 1.65 or 1.45, find the fair price of this option.

[Conclusion]

(1) The risk-neutral probabilities calculated by a dollar trader and a Sterling trader are different.

(2) The dollar cost at time zero of the option valued by either a dollar trader or a Sterling trader is the same.
[Exercise 1.13]

Suppose that \( S_1, S_2 \) have payoffs \((10, -10, 3)\) and \((9, -9, 2)\) in 3 possible scenarios at \( T \).

- If \( S^0 = (1, 1)' \). Is there any arbitrage opportunity?
- If \( S^0 = (1, 0.8)' \). Is there any arbitrage opportunity?

For any asset \( A \) with initial value \( A^0 > 0 \) and payoff \((A^1, \ldots, A^m)\), one way to quantify risk is by the criterion

\[
\text{Risk} = \max_{j=1,\ldots,m} \left| \frac{A^j}{A^0} \right|.
\]

- Find the Risk for the portfolio \( V_a = S_1 \).
- Find the Risk for the portfolio \( V_b = S_1 - S_2 \).
- Can you construct a portfolio using \( S_1 \) and \( S_2 \) with minimum Risk?