1. (30) Fill in the missing values in the following tables of regression output.

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>d.f.</th>
<th>Mean Square</th>
<th>F-statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>9.5e-09</td>
<td></td>
<td></td>
<td></td>
<td>9.5e-09</td>
</tr>
<tr>
<td>Residuals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>s.e.</th>
<th>t-statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.00322</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>-2.04245</td>
<td></td>
<td></td>
<td>0.00322</td>
</tr>
</tbody>
</table>

p.s. It is found in R that $qf(1 - 9.5e^{-9}, 1, 6) = 1917.3$. Also $\bar{x} = 5.125$, $\bar{y} = -9.1974$, $SXX = 54.875$.

2. Let $Y = (9, 4, 1, 3)'$, $X1 = (9, 3, 9, 4)'$, $X2 = (5, 3, 5, 4)'$. It is found that

$$(X'X)^{-1} = \begin{pmatrix} 25 & 2.5 & -9.5 \\ 2.5 & 0.34375 & -1.09375 \\ -9.5 & -1.09375 & 3.84375 \end{pmatrix}$$

i) (10) Fit a regression model $Y = \beta_0 + \beta_1 X1 + \beta_2 X2 + e$. Give the fitted coefficients and estimate of error variance.
ii) (5) Test the significance of $X_1$ on $Y$ after accounting for the effect $X_2$.

iii) (5) Test the significance of $X_1$ on $Y$ without accounting for the effect of $X_2$. 
iv) (10) Test the joint significance of $X_1$ and $X_2$.

v) (5) Construct a 98% confidence interval for $E(Y|X_1 = 6, X_2 = 4)$.

vi) (5) Construct a 90% prediction interval for a new observation with $X_1 = 5, X_2 = 5$. 
vii) (5) Find $\text{var}(\hat{\beta}_0 - \hat{\beta}_2)$.

viii) (5) Using (vii), test $H_0 : \beta_0 = \beta_2$, against $H_A : \beta_0 \neq \beta_2$. 
3. (5) For the regression model

\[ Y = X\beta + e, \]

with \( e \sim N(0, \sigma^2 W) \), where \( W \) is not an identity matrix. A student argues that the ordinary multiple regression cannot be applied, and one should regress \( W^{-1/2}Y \) against \( W^{-1/2}X \) by ordinary multiple regression to estimate \( \beta \). Do you agree with him? If yes, give a formula for the estimator \( \hat{\beta} \) and give the variance of this estimator. Otherwise, explain why he is incorrect and discuss how to estimate \( \beta \).

4. (5) Consider the linear regression model

\[ Y = \beta_0 + \beta_1 X + e. \]

Suppose \( X \) has unit \textit{meter}. If the regression is fit with \( X \) converted to the unit \textit{centimeter}, what will be the changes in

1) \( \hat{\beta}_0 \), 2) \( \hat{\sigma}^2 \), 3) \( \text{var}(\hat{\beta}_1) \), 4) \( t \)-statistic for testing \( \beta_1 = 0 \). 5) F-statistics?
5. (5) $Y, X_1$ and $X_2$ are observations from five subjects. Consider the regression models

1) \[ Y = a_1 + b_1 X_2 + e. \]
2) \[ X_1 = a_2 + b_2 X_2 + e. \]
3) \[ Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + e. \]

The first four residuals from the fitting of model 1) are -0.415, 0.108, -1.13, 0.994. The first four residuals from the fitting of model 2) are -0.373, -0.0868, -1.21, 1.29. Find the estimate of $\beta_1$ for the regression model 3).

6. Given observations $y=(y_1, \ldots, y_n)$, $x=(x_1, \ldots, x_n)$. Let $1=(1, 1, \ldots, 1)$ ($n$-dimensional).

i) (1) Find the residuals, $\hat{e}_1$, for the regression of $y$ against $1$.

ii) (1) Find the residuals, $\hat{e}_II$, for the regression of $x$ against $1$.

iii) (3) Find the intercept and slope estimates for the regression of $\hat{e}_1$ against $\hat{e}_II$. 