Solutions to Revision Exercises

1. Assuming that X and Y are independent. If $X \sim N(0, \sigma^2)$, then $Var(X) = \sigma^2$.

   1) If $X \sim N(0, 4)$ and $Y \sim N(0, 4)$, what is the distribution of $X^2 + Y^2$?
   2) If $X \sim N(0, 1)$ and $Y \sim \chi^2_3$, what is the distribution of $X^2 + Y$?
   3) If $X \sim N(0, 1)$ and $Y \sim \chi^2_{10}$, what is the distribution of $X/\sqrt{Y/10}$?
   4) If $t(v)$ stands for t distribution with $d.f.$ v, then what is $t(\infty)$?
   5) If $X \sim N(2, 4)$ and $Y \sim N(1, 3)$, what is the distribution of $X + 3Y$?

2. Let $x_1, \ldots, x_n$ are **known** real numbers, and $y_1, \ldots, y_n$ are independent **random variables** with mean 0 and variance 1. Define $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$ and $\bar{y} = \frac{\sum_{i=1}^n y_i}{n}$.

   6) Find $\sum_{i=1}^n (y_i - \bar{y})$.
   7) Find $\sum_{i=1}^n (y_i - \bar{y})^2 - \sum_{i=1}^n y_i^2 - n\bar{y}^2$.
   8) Find $Var(\sum_{i=1}^n \sqrt{i} y_i)$
   9) Define $SXX = \sum_{i=1}^n (x_i - \bar{x})^2$. Find $Var(\sum_{i=1}^n \frac{x_i - \bar{x}}{SXX} y_i)$

3. Let

$$M = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}. \qquad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}.$$

   10) Let $f(\beta) = \beta' M \beta$. Find $\frac{\partial f(\beta)}{\partial \beta}$.
   11) Let $g(\beta) = (1\ 1\ 1) M^{-1} \beta$. Find $\frac{\partial g(\beta)}{\partial \beta}$.
   12) Let $h(\beta) = \beta' \beta$. Find the minimum value of the function h.

4. Let X be a $n \times p$-dimensional matrix, Y be a n-dimensional vector, β be a p-dimensional vector. Let $H = X(X'X)^{-1}X'$.

   13) Solve for β from $X'(Y - X\beta) = 0$.
   14) Find $(I - H)X$, where I is the identity matrix.
   15) Is H symmetric?
   16) Find H^{3008}.

01. $\underline{4\chi^2_2}$ 02. $\underline{\chi^2_4}$ 03. $\underline{t_{10}}$ 04. $\underline{N(0, 1)}$ 05. $\underline{0}$ 06. $\underline{0}$ 07. $\underline{-2n\bar{y}^2 \text{ or } 0}$ 08. $\underline{n(n+1)/2}$ 09. $\underline{1/SXX}$

10. $\underline{\beta = \begin{pmatrix} 2\beta_1 + 4\beta_2 \\ 4\beta_1 + 4\beta_2 \\ 8\beta_3 \end{pmatrix}}$ 11. $\underline{\beta = \begin{pmatrix} -1 \\ 2 \\ 1/4 \end{pmatrix}}$ 12. $\underline{0}$ 13. $\underline{(X'X)^{-1}X'Y}$ 14. $\underline{0}$ 15. Yes 16. $\underline{H}$