Revision Exercises

1. Assuming that X and Y are independent. If $X \sim N(0, \sigma^2)$, then $Var(X) = \sigma^2$.

    1) If $X \sim N(0, 4)$ and $Y \sim N(0, 4)$, what is the distribution of $X^2 + Y^2$?

    2) If $X \sim N(0, 1)$ and $Y \sim \chi_3^2$, what is the distribution of $X^2 + Y$?

    3) If $X \sim N(0, 1)$ and $Y \sim \chi_{10}^2$, what is the distribution of $X/\sqrt{Y/10}$?

    4) If $t(v)$ stands for t distribution with $d.f.$ v, then what is $t(\infty)$?

    5) If $X \sim N(2, 4)$ and $Y \sim N(1, 3)$, what is the distribution of $X + 3Y$?

2. Let $x_1, \ldots, x_n$ are **known** real numbers, and $y_1, \ldots, y_n$ are independent **random variables** with mean 0 and variance 1. Define $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ and $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$.

    6) Find $\sum_{i=1}^{n}(y_i - \bar{y})$.

    7) Find $\sum_{i=1}^{n}(y_i - \bar{y})^2 - \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$.

    8) Find $Var(\sum_{i=1}^{n} \sqrt{i} y_i)$

    9) Define $SXX = \sum_{i=1}^{n}(x_i - \bar{x})^2$. Find $Var(\sum_{i=1}^{n} \frac{x_i - \bar{x}}{SXX} y_i)$

3. Let
$$M = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}. \qquad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}.$$

    10) Let $f(\beta) = \beta' M \beta$. Find $\frac{\partial f(\beta)}{\partial \beta}$.

    11) Let $g(\beta) = (1\ 1\ 1)M^{-1}\beta$. Find $\frac{\partial g(\beta)}{\partial \beta}$.

    12) Let $h(\beta) = \beta' \beta$. Find the minimum value of the function h.

4. Let X be a $n \times p$-dimensional matrix, Y be a n-dimensional vector, β be a p-dimensional vector. Let $H = X(X'X)^{-1}X'$.

    13) Solve for β from $X'(Y - X\beta) = 0$.

    14) Find $(I - H)X$, where I is the identity matrix.

    15) Is H symmetric?

    16) Find H^{3008}.

01. _____ 02. _____ 03. _____ 04. _____ 05. _____ 06. _____

07. _____ 08. _____ 09. _____ 10. _____ 11. _____ 12. _____

13. _____ 14. _____ 15. _____ 16. _____