STAT 3008 Applied Regression Analysis
Midterm
13:30-15:15. Thursday, 16 Oct 2014

Name: ______________________ **Major:** ______________________

Part A: Circle the correct answer. (50 Marks)

1. The regression equation for predicting number of speeding tickets (Y) from information about driver age (X) is Y = -0.065(X) + 5.57. How many tickets would you predict for a twenty-year-old?

   a) 6
   b) 4.27
   c) 5.57
   d) 1

2. A regression equation was computed to be Y = 35 + 6X. The value of 35 indicates that

   a) An increase in one unit of X will result in an increase of 35 in Y
   b) The coefficient of correlation is 35
   c) The coefficient of determination is 35
   d) The regression line crosses the Y-axis at 35

3. Null plot means a scatter plot with what characteristic(s)?
   (i) mean zero
   (ii) constant variance
   (iii) no separated point
   (iv) several leverage points

   a) i and ii
   b) ii, iii and iv
   c) i, ii and iii
   d) all

4. Consider the simple linear model with zero intercept term: $y_i = \beta_1 x_i + e_i$, where $e_i \overset{i.i.d.}{\sim} N(0, \sigma^2)$, which of the following statement(s) is(are) true?
   (i) the least square estimator of coefficient of x is unbiased
   (ii) the expected value of residual is non-zero
   (iii) the coefficient of determination in this model is larger than that in the simple linear model with unfixed intercept term

   a) i
   b) i and ii
   c) i and iii
   d) all

5. CHAN Tai Man collected a data set of size $n = 5$, namely, $\{x_i, y_i\}_{i\in\{1,2,3,4,5\}}$, where y_i denotes the i-th student's GPA while x_i denote his/her I.Q. score. CHAN is a student of Professor YAU, and asked him for help. Prof. YAU suggested that a simple linear regression model may be applied i.e. $Y_i = \beta_0 + \beta_1 x_i + e_i$. You are given that $\{\hat{e}_1, \ldots, \hat{e}_4\} = \{0.1, -0.2, 0.3, -0.1\}$ and $SYY = 3.3$ Find the value of R^2.

   a) 0.9545
   b) 0.9515
   c) 0.0485
   d) 0.9538

6. For the multiple regression model $Y = X\beta + e$, $e \sim N(0, \sigma^2)$, Let $\hat{\beta} = (X'X + kI)^{-1}X'Y$. What is $Var(\hat{\beta})$?

   a) $\sigma^2(X'X + kI)^{-1} - k\sigma^2(X'X + kI)^{-2}$
   b) $\sigma^2(X'X + kI)^{-1}$
   c) $(X'X)^{-1}\sigma^2$
   d) $I\sigma^2$

7. For the regression model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + e, e \sim N(0, \sigma^2 I)$,find $E(\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2)$.

   a) $(n - 3)\sigma^2$
   b) $(n - 2)\sigma^2$
   c) $(n - 1)\sigma^2$
   d) $(n - 4)\sigma^2$

8. In a multiple regression study with 100 observations, an analyst is considering whether or not to include the fourth independent variable to the original model that has only three independent variables. The RSS of the model when the fourth variable is included is 38450. The RSS of the model when the fourth variable is not included is 40000. Based on the partial $F-$statistic of ______ we would conclude that the fourth variable _____ the model at 95 % confidence level.

   a) 3.83; does not significantly improve
   b) 3.87; does not significantly improve
   c) 3.99; significantly improves
   d) 4.03; significantly improves

9. Which is correct about the RSS in simple linear regression?

   a) $RSS = SYY - \frac{SXY^2}{SXX}$
   b) $RSS = SXX - \frac{SXY^2}{SYY}$
   c) $RSS = n\hat{\sigma}^2$
   d) $RSS = (n - 1)\hat{\sigma}^2$

10. Which of the following is a stronger correlation than -0.54?

 a) 0
 b) -0.1
 c) -1
 d) 0.5

11. A teacher examined the relationship between the number of hours devoted to reading each week Y and the predictor variables social class X_1, number of years of school completed X_2, and reading speed measured by pages read per hourX_3. $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$. A sample is recorded. The sequential ANOVA table, as terms entering the model from top to bottom, is shown.

	Df	$SumSq$
X_3	1	1058.63
X_2	1	183.74
X_1	1	37.99
$Residuals$	15	363.30

Based on the above table, can we do the test $H_0 : \beta_1 = \beta_2 = 0, \beta_0, \beta_3$ arbitrary against $H_1 : \beta_i$arbitrary for all i ? If can, what is the F-statistic and the result?

 a) Yes, 4.577, Reject H_0
 b) Yes, 4.577, Cannot Reject H_0
 c) Yes, 6.3589, Reject H_0
 d) No

12. Which is a multiple regression?

 a) $y = \beta_o + \beta_1 x$
 b) $y = \beta_o + \beta_1 x + \epsilon$
 c) $y = \beta_o + \epsilon$
 d) $y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

13. If β is an $n \times 1$ vector and $\mathbf{M}$ is an $n \times n$ matrix, then what should $\frac{\partial}{\partial \beta} \beta' \mathbf{M} \beta$?

 a) $\mathbf{M} \times \beta$
 b) $\mathbf{M}' \times \beta$
 c) $\mathbf{M}' \times \mathbf{M}$
 d) $(\mathbf{M}' + \mathbf{M}) \times \beta$

14. What is wrong about the error term e in simple linear regression?

 a) $Var(e) = Var(Y)$
 b) $Var(\hat{e}) = \sigma^2$
 c) e is i.i.d
 d) $E(\hat{e}) = E(e)$

15. Which of the following has the CORRECT form for the RSS in simple linear regression?

a) $\sum \hat{e}_i$
b) $SYY - \frac{SXY^2}{SYY}$
c) $SYY - \hat{\beta_1}^2 SXX$
d) $\sigma^2(n-2)$

16. Consider a situation in which the regression data set is divided into two groups, given by $y_i = \beta_{01} + \beta_1(x_i - \bar{x}_1) + \epsilon_i$ for i=1,2 $\ldots n_1$ and $y_i = \beta_{02} + \beta_1(x_i - \bar{x}_2) + \epsilon_i$ for i=$n_1+1, n_1+2, \ldots n_1+n_2$ where $\bar{x}_1 = \sum_{i=1}^{n_1} \frac{x_i}{n_1}$ and $\bar{x}_2 = \sum_{i=n_1+1}^{n_1+n_2} \frac{x_i}{n_2}$

Let $A = \sum_{i=1}^{n_1}(x_i - \bar{x}_1)y_i$,$B = \sum_{i=n_1+1}^{n_1+n_2}(x_i - \bar{x}_2)y_i$,$C = \sum_{i=1}^{n_1}(x_i - \bar{x}_1)^2$and $D = \sum_{i=n_1+1}^{n_1+n_2}(x_i - \bar{x}_2)^2$. Derive the OLS estimate for β_1.

a) $\frac{(A)}{(C)}$
b) $\frac{(A+B)}{(C+D)}$
c) $\frac{(B)}{(D)}$
d) $\frac{(A+C)}{(B+D)}$

17. Which of the following is the unbiased estimation of σ^2 as in the simple linear regression model?

a) $\frac{\sum \hat{e_i}^2}{n-2}$
b) $\frac{RSS}{n}$
c) $\frac{SXX - \frac{SXY^2}{SXX}}{n-2}$
d) $\frac{SXX - \frac{SXX^2}{SXY}}{n-2}$

18. Which of the following statements about error and residual is(are) correct?
(i) error is the vertical distance between observed response and the true response
(ii) residual is the vertical distance between observed response and the predicted value of response
(iii) error is deterministic but residual is random

a) i
b) i and ii
c) ii
d) all

19. For a simple linear regression model,

$$Y = \beta_0 + \beta_1 x + e \qquad e_i \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma^2\right)$$

Given the following X observations $\mathcal{A}_X = \{5, 3, 5, 4\}$ and Y observations $\mathcal{A}_Y = \{9, 3, 9, 4\}$. If Helen wants to test the null hypothesis against the alternative hypothesis at 10% significance level,

$$H_0 : \beta_0 = 0 \qquad H_1 : \beta_0 < 0$$

then what are the correct p-value and conclusion?

a) $p = 0.2994$ and cannot reject Null
b) $p = 0.1472$ and cannot reject Null
c) $p = 0.0736$ and reject Null
d) $p = 0.1472$ and reject Null

20. For the multiple regression model $Y = X\beta + e$, $e \sim N(0, \sigma^2)$, Let $\hat{\beta} = (X'X + kI)^{-1}X'Y$. What is $E(\hat{\beta})$?

a) β
b) $\beta - k(X'X + kI)^{-1}\beta$
c) $\beta - k(X'X + kI)\beta$
d) $\beta - kX'X\beta$

Part B: Write down the calculation steps and answers on the blank area. (50 Marks)

1. (5 marks) In a multiple regression analysis, MSE = 30, number of observation = 58, number of predictor = 4, and TSS = 2000. What is the R^2 of the regression?

2. (20 marks) Consider multiple regression $Y = X\beta + e$, where Y, e are $n \times 1$ and X is $n \times p$. Let J be a squared matrix of size n with all the entries equal $1/n$. and $H = X(X'X)^{-1}X'$. Let Y_i and $\hat{Y}_i$ be the i^{th} observation and its fitted value. Recall that $RSS = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$ and $SYY = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$, where $\bar{Y} = \sum_{i=1}^{n} Y_i/n$ is the sample mean.

   i) (3 marks) What is JY?

   ii) (3 marks) Show that $RSS = (Y - HY)'(Y - HY) = Y'(I - H)Y$.

   iii) (3 marks) Show that $SYY = (Y - JY)'(Y - JY) = Y'(I - J)Y$.

iv) (3 marks) Suppose that the first column of X is $(1, 1, \ldots, 1)'$. Show that $JH = HJ = J$.

v) (2 marks) We define $SSreg = \sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2 =$ because this is the "sum of square" of the "regression sample" $\{\hat{Y}_1, \ldots, \hat{Y}_n\}$. Show that $\bar{Y} = \sum_{i=1}^{n} \hat{Y}_n / n$.

vi) (3 marks) Using (iv) and (v), show that $SSreg = (HY - JY)'(HY - JY) = Y'(H - J)Y$.

vii) (3 marks) Using (ii), (iii) and (vi), show the decomposition of sum of squares: $SYY = RSS + SSreg$.

3. (25 marks) The results of thirteen countries in the 2014 Asian Olympic game is given below:

Country	Gold (G)	Silver (S)	Bronze (B)
China	151	108	83
Korea	79	71	84
Japan	47	76	77
Kazakhstan	28	23	33
Iran	21	18	18
Thailand	12	7	28
North Korea	11	11	14
India	11	10	36
Taiwan	10	18	23
Qatar	10	0	4
Uzbekistan	9	14	21
Bahrain	9	6	4
Hong Kong	6	12	24

i) (5 marks) Plot a scatterplot of G against S. Which country would have the most influence on the regression fit?

ii) (5 marks) It is found that

$$\frac{1}{n}\sum G_i = 31.1,\ \frac{1}{n}\sum S_i = 28.8,\ \frac{1}{n}\sum G_i^2 = 2558.5,\ \frac{1}{n}\sum S_i^2 = 1869.5,\ \frac{1}{n}\sum S_i G_i = 2096.8$$

Use these results to estimate the regression model $G = \beta_0 + \beta_1 S + e$, $e \sim N(0, \sigma^2)$. Plot this line on the above scatter plot diagram. Is this fitting appropriate?

iii) (5 marks) Is there linear relationship between the number of gold and silver medals at 5% significance level?

iv) (5 marks) Write down the equation governing the 95% confidence ellipse for the parameter (β_0, β_1).

v) (5 marks) If Country A got 20 silver medals, what is the 90% prediction interval of its number of gold medal?

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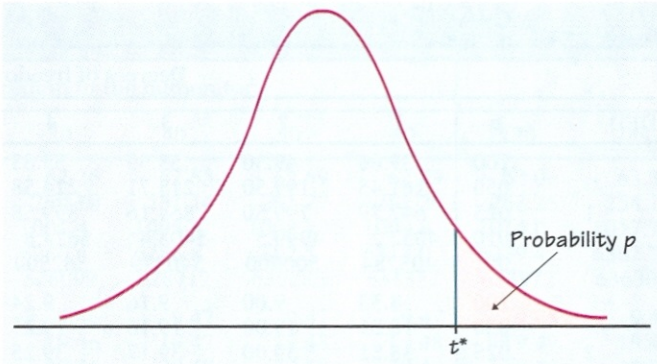

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

TABLE D *t* distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table 10 F distribution — inverse cdf

							df_1								
df_2	1	2	3	4	5	6	7	8	10	12	24	60	120	∞	
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	241.9	243.9	249.1	252.2	253.3	254.3	0.95
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.6	968.6	976.7	997.3	1010	1014	1018.3	0.975
1	4052	4999	5404	5624	5764	5859	5928	5981	6056	6107	6234	6313	6340	6366.0	0.99
1	405K	500K	540K	563K	576K	586K	593K	598K	606K	610K	624K	631K	634K	637K	0.999
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.40	19.41	19.45	19.48	19.49	19.50	0.95
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.40	39.41	39.46	39.48	39.49	39.50	0.975
2	98.50	99.00	99.16	99.25	99.30	99.33	99.36	99.38	99.40	99.42	99.46	99.48	99.49	99.50	0.99
2	998.4	998.8	999.3	999.3	999.3	999.3	999.3	999.3	999.3	999.3	999.3	999.3	999.3	999.3	0.999
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.785	8.745	8.638	8.572	8.549	8.526	0.95
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.42	14.34	14.12	13.99	13.95	13.90	0.975
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.23	27.05	26.60	26.32	26.22	26.13	0.99
3	167.1	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.2	128.3	125.9	124.4	124.0	123.5	0.999
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.964	5.912	5.774	5.688	5.658	5.628	0.95
4	12.218	10.649	9.979	9.604	9.364	9.197	9.074	8.980	8.844	8.751	8.511	8.360	8.309	8.257	0.975
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.55	14.37	13.93	13.65	13.56	13.46	0.99
4	74.13	61.25	56.17	53.43	51.72	50.52	49.65	49.00	48.05	47.41	45.77	44.75	44.40	44.05	0.999
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.735	4.678	4.527	4.431	4.398	4.365	0.95
5	10.01	8.434	7.764	7.388	7.146	6.978	6.853	6.757	6.619	6.525	6.278	6.123	6.069	6.015	0.975
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.05	9.888	9.466	9.202	9.112	9.020	0.99
5	47.18	37.12	33.20	31.08	29.75	28.83	28.17	27.65	26.91	26.42	25.13	24.33	24.06	23.78	0.999
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.060	4.000	3.841	3.740	3.705	3.669	0.95
6	8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.461	5.366	5.117	4.959	4.904	4.849	0.975
6	13.75	10.92	9.780	9.148	8.746	8.466	8.260	8.102	7.874	7.718	7.313	7.057	6.969	6.880	0.99
6	35.51	27.00	23.71	21.92	20.80	20.03	19.46	19.03	18.41	17.99	16.90	16.21	15.98	15.75	0.999
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.637	3.575	3.410	3.304	3.267	3.230	0.95
7	8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.761	4.666	4.415	4.254	4.199	4.142	0.975
7	12.25	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.620	6.469	6.074	5.824	5.737	5.650	0.99
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.08	13.71	12.73	12.12	11.91	11.70	0.999
8	5.318	4.459	4.066	3.838	3.688	3.581	3.500	3.438	3.347	3.284	3.115	3.005	2.967	2.928	0.95
8	7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.295	4.200	3.947	3.784	3.728	3.670	0.975
8	11.26	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.814	5.667	5.279	5.032	4.946	4.859	0.99
8	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.54	11.19	10.30	9.728	9.532	9.333	0.999
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.137	3.073	2.900	2.787	2.748	2.707	0.95
9	7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	3.964	3.868	3.614	3.449	3.392	3.333	0.975
9	10.56	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.257	5.111	4.729	4.483	4.398	4.311	0.99
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	9.894	9.570	8.724	8.186	8.002	7.813	0.999
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	2.978	2.913	2.737	2.621	2.580	2.538	0.95
10	6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.717	3.621	3.365	3.198	3.140	3.080	0.975
10	10.04	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.849	4.706	4.327	4.082	3.996	3.909	0.99
10	21.04	14.90	12.55	11.28	10.48	9.926	9.517	9.204	8.754	8.446	7.638	7.122	6.944	6.763	0.999
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.854	2.788	2.609	2.490	2.448	2.404	0.95
11	6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.526	3.430	3.173	3.004	2.944	2.883	0.975
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.539	4.397	4.021	3.776	3.690	3.602	0.99
11	19.69	13.81	11.56	10.35	9.579	9.047	8.655	8.355	7.923	7.625	6.848	6.348	6.175	5.999	0.999
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.753	2.687	2.505	2.384	2.341	2.296	0.95
12	6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.374	3.277	3.019	2.848	2.787	2.725	0.975
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.296	4.155	3.780	3.535	3.449	3.361	0.99
12	18.64	12.97	10.80	9.633	8.892	8.378	8.001	7.711	7.292	7.005	6.249	5.763	5.593	5.420	0.999
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.602	2.534	2.349	2.223	2.178	2.131	0.95
14	6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	3.147	3.050	2.789	2.614	2.552	2.487	0.975
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	3.939	3.800	3.427	3.181	3.094	3.004	0.99
14	17.14	11.78	9.730	8.622	7.922	7.436	7.078	6.802	6.404	6.130	5.407	4.938	4.773	4.604	0.999

	df_1														
df_2	1	2	3	4	5	6	7	8	10	12	24	60	120	∞	
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.494	2.425	2.235	2.106	2.059	2.010	0.95
16	6.115	4.687	4.077	3.729	3.502	3.341	3.219	3.125	2.986	2.889	2.625	2.447	2.383	2.316	0.975
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.890	3.691	3.553	3.181	2.933	2.845	2.753	0.99
16	16.12	10.97	9.006	7.944	7.272	6.805	6.460	6.195	5.812	5.547	4.846	4.388	4.226	4.059	0.999
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.412	2.342	2.150	2.017	1.968	1.917	0.95
18	5.978	4.560	3.954	3.608	3.382	3.221	3.100	3.005	2.866	2.769	2.503	2.321	2.256	2.187	0.975
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.508	3.371	2.999	2.749	2.660	2.566	0.99
18	15.38	10.39	8.487	7.460	6.808	6.355	6.021	5.763	5.390	5.132	4.447	3.996	3.836	3.670	0.999
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.348	2.278	2.082	1.946	1.896	1.843	0.95
20	5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.774	2.676	2.408	2.223	2.156	2.085	0.975
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.368	3.231	2.859	2.608	2.517	2.421	0.99
20	14.82	9.953	8.098	7.096	6.461	6.019	5.692	5.440	5.075	4.823	4.149	3.703	3.544	3.378	0.999
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.255	2.183	1.984	1.842	1.790	1.733	0.95
24	5.717	4.319	3.721	3.379	3.155	2.995	2.874	2.779	2.640	2.541	2.269	2.080	2.010	1.935	0.975
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.168	3.032	2.659	2.403	2.310	2.211	0.99
24	14.03	9.340	7.554	6.589	5.977	5.551	5.235	4.991	4.638	4.393	3.735	3.295	3.136	2.969	0.999
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.165	2.092	1.887	1.740	1.683	1.622	0.95
30	5.568	4.182	3.589	3.250	3.026	2.867	2.746	2.651	2.511	2.412	2.136	1.940	1.866	1.787	0.975
30	7.562	5.390	4.510	4.018	3.699	3.473	3.305	3.173	2.979	2.843	2.469	2.208	2.111	2.006	0.99
30	13.29	8.773	7.054	6.125	5.534	5.122	4.817	4.582	4.239	4.001	3.357	2.920	2.760	2.589	0.999
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.077	2.003	1.793	1.637	1.577	1.509	0.95
40	5.424	4.051	3.463	3.126	2.904	2.744	2.624	2.529	2.388	2.288	2.007	1.803	1.724	1.637	0.975
40	7.314	5.178	4.313	3.828	3.514	3.291	3.124	2.993	2.801	2.665	2.288	2.019	1.917	1.805	0.99
40	12.61	8.251	6.595	5.698	5.128	4.731	4.436	4.207	3.874	3.643	3.011	2.574	2.410	2.233	0.999
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.026	1.952	1.737	1.576	1.511	1.438	0.95
50	5.340	3.975	3.390	3.054	2.833	2.674	2.553	2.458	2.317	2.216	1.931	1.721	1.639	1.545	0.975
50	7.171	5.057	4.199	3.720	3.408	3.186	3.020	2.890	2.698	2.563	2.183	1.909	1.803	1.683	0.99
50	12.22	7.956	6.336	5.459	4.901	4.512	4.222	3.998	3.671	3.443	2.817	2.378	2.211	2.026	0.999
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	1.993	1.917	1.700	1.534	1.467	1.389	0.95
60	5.286	3.925	3.343	3.008	2.786	2.627	2.507	2.412	2.270	2.169	1.882	1.667	1.581	1.482	0.975
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.632	2.496	2.115	1.836	1.726	1.601	0.99
60	11.97	7.768	6.171	5.307	4.757	4.372	4.086	3.865	3.542	3.315	2.694	2.252	2.082	1.890	0.999
80	3.960	3.111	2.719	2.486	2.329	2.214	2.126	2.056	1.951	1.875	1.654	1.482	1.411	1.325	0.95
80	5.218	3.864	3.284	2.950	2.730	2.571	2.450	2.355	2.213	2.111	1.820	1.599	1.508	1.400	0.975
80	6.963	4.881	4.036	3.563	3.255	3.036	2.871	2.742	2.551	2.415	2.032	1.746	1.630	1.494	0.99
80	11.67	7.540	5.972	5.123	4.582	4.204	3.923	3.705	3.386	3.162	2.545	2.099	1.924	1.720	0.999
100	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032	1.927	1.850	1.627	1.450	1.376	1.283	0.95
100	5.179	3.828	3.250	2.917	2.696	2.537	2.417	2.321	2.179	2.077	1.784	1.558	1.463	1.347	0.975
100	6.895	4.824	3.984	3.513	3.206	2.988	2.823	2.694	2.503	2.368	1.983	1.692	1.572	1.427	0.99
100	11.50	7.408	5.857	5.017	4.482	4.107	3.829	3.612	3.296	3.074	2.458	2.009	1.829	1.615	0.999
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.910	1.834	1.608	1.429	1.352	1.254	0.95
120	5.152	3.805	3.227	2.894	2.674	2.515	2.395	2.299	2.157	2.055	1.760	1.530	1.433	1.310	0.975
120	6.851	4.787	3.949	3.480	3.174	2.956	2.792	2.663	2.472	2.336	1.950	1.656	1.533	1.381	0.99
120	11.38	7.321	5.781	4.947	4.416	4.044	3.767	3.552	3.237	3.016	2.402	1.950	1.767	1.543	0.999
∞	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.831	1.752	1.517	1.318	1.221	1.000	0.95
∞	5.024	3.689	3.116	2.786	2.566	2.408	2.288	2.192	2.048	1.945	1.640	1.388	1.268	1.000	0.975
∞	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.321	2.185	1.791	1.473	1.325	1.000	0.99
∞	10.83	6.908	5.422	4.617	4.103	3.743	3.474	3.265	2.959	2.742	2.132	1.660	1.447	1.000	0.999