

**STAT 3008**  
**Exercises 3**

**Problems** refer to the problem sets in the textbook: Applied Linear Regression, 3rd edition by Weisberg.

1. (i) Problem 2.4.2.  
(ii) Problem 2.4.3.  
(iii) From the regression  $E(Dheight|Mheight) = \beta_o + \beta_1 Mheight$ , express the relation in the form  $Mheight = \alpha_o + \alpha_1 E(Dheight|Mheight)$ .  
(iv) Fit the model  $E(Mheight|Dheight) = \beta_o + \beta_1 Dheight$ . Is it the same as (iii)?

**Remarks.** Part (iii) and (iv) show that regression treats  $x$  and  $y$  differently. Note that  $\hat{\beta}_1 < 1$  no matter  $Mheight$  is chosen to be  $x$  or  $y$ . This is an example of **regression to the mean**. The mathematical reason is that  $SXX \approx SY Y$ , so no matter how you do the regression, you have  $\hat{\beta}_1 = SXY/SXX$  or  $SXY/SYY$ , both are  $\approx SXY/\sqrt{SXX SY Y} = r_{xy} < 1$ .

2. Problem 2.7.
3. Problem 2.8.
4. Problem 2.10.1 and 2.10.2.
5. Problem 2.12.
6. Show that  $Cov(\bar{y}, \hat{\beta}_1) = 0$ .
7. Let

$$X = \begin{pmatrix} 1 & 3 \\ 1 & 2 \\ 1 & 5 \\ 1 & 1 \\ 1 & 2 \\ 1 & 8 \\ 1 & 0 \end{pmatrix}.$$

Find  $X'X$ ,  $XX'$ ,  $(X'X)^{-1}$ ,  $tr(X'X)$  and  $tr(XX')$ .