STAT 3008
Exercises 2

Problems refer to the problem sets in the textbook: Applied Linear Regression, 3rd edition by Weisberg.

1. Problem 2.1. For 2.1.3, no need to do t-tests.

2. Problem 2.3.

3. Problem 2.4.1.

4. This problem shows the unbiasedness of the error estimate, $\hat{\sigma}^2$.

   a) Show that

   $$y_i - \bar{y} = \beta_1(x_i - \bar{x}) + e_i - \bar{e}\,,$$

   where $\bar{e} = \sum_{i=1}^n e_i/n$.

   b) Using a), show that

   $$\mathrm{E}\sum_{i=1}^n (y_i - \bar{y})^2 = \beta_1^2 \sum_{i=1}^n (x_i - \bar{x})^2 + (n-1)\sigma^2\,.$$

   c) Show that

   $$\sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n (y_i - \bar{y}_i)^2 - \hat{\beta}_1{}^2 \sum_{i=1}^n (x_i - \bar{x})^2\,.$$

   d) Using b),c) and the formula for $Var(\hat{\beta}_1)$, show that

   $$E(\hat{\sigma}^2) := E\left(\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}\right) = \sigma^2\,.$$