STAT 3008 Applied Regression Analysis

Assignment2 Solution

5 Oct. 2013

1.

$$\begin{aligned}
\sum_{i=1}^{n}(y_i-\bar{y})^2 &= \sum_{i=1}^{n}(y_i-\hat{y}_i+\hat{y}_i-y)^2 \\
&= \sum_{i=1}^{n}(y_i-\hat{y}_i)^2+\sum_{i=1}^{n}(y_i-\bar{y})^2+2\sum_{i=1}^{n}(y_i-\hat{y}_i)(\hat{y}_i-\bar{y}) \\
&= \sum_{i=1}^{n}(y_i-\hat{y}_i)^2+\sum_{i=1}^{n}(y_i-\bar{y})^2+2\sum_{i=1}^{n}\hat{e}_i(\hat{y}_i-\bar{y}) \\
&= \sum_{i=1}^{n}(y_i-\hat{y}_i)^2+\sum_{i=1}^{n}(y_i-\bar{y})^2+2\sum_{i=1}^{n}\hat{e}_i\hat{y}_i-2\bar{y}\sum_{i=1}^{n}\hat{e}_i \\
&= \sum_{i=1}^{n}(y_i-\hat{y}_i)^2+\sum_{i=1}^{n}(y_i-\bar{y})^2 \\
&\quad (\because \sum_{i=1}^{n}\hat{e}_i\hat{y}_i=0\ \&\ \sum_{i=1}^{n}\hat{e}_i=0)
\end{aligned}$$

2.(i)

$$RSS(\beta)=\sum_{i=1}^{n}(y_i-\beta-\beta x_i)^2$$

$$\frac{dRSS}{d\beta}=-2\sum_{i=1}^{n}(y_i-\beta-\beta x_i)(1+x_i)$$

$$\frac{dRSS}{d\beta}|_{\beta=\hat{\beta}}=0$$

$$\sum_{i=1}^{n}(y_i-\beta-\beta x_i)(1+x_i)=0$$

$$\therefore \hat{\beta}=\frac{\sum_{i=1}^{n}(1+x_i)y_i}{\sum_{i=1}^{n}(1+x_i)^2}.$$

$$\therefore \hat{\sigma}^2=\frac{R\hat{S}S}{n-1}=\frac{\sum_{i=1}^{n}\hat{e}_i^2}{n-1}$$

(ii)

$$
\begin{aligned}
E(\hat{\beta}|X) &= E[\frac{\sum_{i=1}^n (1+x_i)y_i}{\sum_{i=1}^n (1+x_i)^2}|X] \\
&= \frac{\sum_{i=1}^n (1+x_i)E(y_i|X)}{\sum_{i=1}^n (1+x_i)^2} \\
&= \frac{\sum_{i=1}^n (1+x_i)(\beta+\beta x_i)}{\sum_{i=1}^n (1+x_i)^2} \\
&= \frac{\sum_{i=1}^n (1+x_i)^2\beta}{\sum_{i=1}^n (1+x_i)^2} \\
&= \beta
\end{aligned}
$$

$$
\begin{aligned}
Var(\hat{\beta}|X) &= Var[\frac{\sum_{i=1}^n (1+x_i)y_i}{\sum_{i=1}^n (1+x_i)^2}|X] \\
&= \frac{\sum_{i=1}^n (1+x_i)^2 Var(y_i|X)}{[\sum_{i=1}^n (1+x_i)^2]^2} \\
&= \frac{\sum_{i=1}^n (1+x_i)^2\sigma^2}{[\sum_{i=1}^n (1+x_i)^2]^2} \\
&= \frac{\sigma^2}{\sum_{i=1}^n (1+x_i)^2}
\end{aligned}
$$

(iii)

$$
E(\hat{\sigma}^2) = E[\frac{\sum_{i=1}^n \hat{e}_i^2}{n-1}] = \frac{\sum_{i=1}^n E[\hat{e}_i^2]}{n-1}
$$

$$
\begin{aligned}
E[\hat{e}_i^2] &= Var(\hat{e}_i) + [E(\hat{e}_i)]^2 \\
&= Var(\hat{e}_i) \quad (\because \quad E(\hat{e}_i) = E(y_i-\hat{y}_i) = E(y_i) - E[\hat{\beta}(1+x_i)] = 0 \\
&= Var(y_i-\hat{y}_i) \\
&= Var(y_i) + Var(\hat{y}_i) - 2Cov(y_i,\hat{y}_i)
\end{aligned}
$$

$$Var(y_i) = \sigma^2,$$

$$Var(\hat{y}_i) = Var[\hat{\beta}(1+x_i)] = (1+x_i)^2 Var(\hat{\beta}) = \frac{(1+x_i)^2\sigma^2}{\sum_{i=1}^n (1+x_i)^2}.$$

$$
\begin{aligned}
Cov(y_i,\hat{y}_i) &= E(y_i\hat{y}_i) - E(y_i)E(\hat{y}_i) \\
&= E[y_i(1+x_i)\hat{\beta}] - [(1+x_i)\beta][(1+x_i)\beta] \\
&= (1+x_i)E[y_i\frac{\sum_{j=1}^n (1+x_j)y_j}{\sum_{j=1}^n (1+x_j)^2}] - \beta^2(1+x_i)^2 \\
&= \frac{1+x_i}{\sum_{j=1}^n (1+x_j)^2} E[(1+x_i)y_i^2 + y_i\sum_{j\neq i}(1+x_j)y_j] - \beta^2(1+x_i)^2 \\
&= \frac{1+x_i}{\sum_{j=1}^n (1+x_j)^2} [(1+x_i)E(y_i^2) + \sum_{j\neq i}(1+x_j)E(y_iy_j)] - \beta^2(1+x_i)^2
\end{aligned}
$$

$\because \quad E(y_i^2) = Var(y_i) + [E(y_i)]^2 = \sigma^2 + [\beta(1+x_i)]^2 = \sigma^2 + \beta^2(1+x_i)^2,$
and for $j \neq i$,

$$E(y_iy_j) = E(y_i)E(y_j) + Cov(y_i, y_j)$$
$$= E(y_i)E(y_j) \quad (\because\ y_i, y_j \text{ are independent.})$$
$$= \beta(1+x_i)\beta(1+x_j)$$
$$= \beta^2(1+x_i)(1+x_j)$$

$$= \frac{(1+x_i)^2}{\quad} \{\sigma^2 + \beta^2(1+x_i)^2 + \sum(1+x_j)^2\beta^2\} - \beta^2(1+x_i)^2$$

$$\therefore\ Cov(y_i, \hat{y}_i) = \frac{1+x_i}{\sum_{j=1}^{n}(1+x_j)^2}\{(1+x_i)[\sigma^2 + \beta^2(1+x_i)^2] + \sum_{j\neq i}(1+x_j)[\beta^2(1+x_i)(1+x_j)])\}$$
$$-\beta^2(1+x_i)^2$$
$$= \frac{\sigma^2(1+x_i)^2}{\sum_{j=1}^{n}(1+x_j)^2} + \beta^2(1+x_i)^2 - \beta^2(1+x_i)^2$$
$$= \frac{\sigma^2(1+x_i)^2}{\sum_{j=1}^{n}(1+x_j)^2}$$

$$\therefore\ E[\hat{e}_i^2] = \sigma^2 + \frac{(1+x_i)^2\sigma^2}{\sum_{i=1}^{n}(1+x_i)^2} - 2\frac{\sigma^2(1+x_i)^2}{\sum_{j=1}^{n}(1+x_j)^2}$$
$$= \sigma^2 - \frac{(1+x_i)^2\sigma^2}{\sum_{i=1}^{n}(1+x_i)^2}$$

$$\therefore\ \sum_{i=1}^{n} E[\hat{e}_i^2] = \sum_{i=1}^{n}\{\sigma^2 - \frac{(1+x_i)^2\sigma^2}{\sum_{i=1}^{n}(1+x_i)^2}\}$$
$$= n\sigma^2 - \frac{\sum_{i=1}^{n}(1+x_i)^2\sigma^2}{\sum_{i=1}^{n}(1+x_i)^2}$$
$$= n\sigma^2 - \sigma^2$$
$$= (n-1)\sigma^2$$

$$\therefore\ E(\hat{\sigma}^2) = \sigma^2$$

(iv)

$$\because\ E(\hat{\beta}|X) = \beta, Var(\hat{\beta}|X) = \frac{\sigma^2}{\sum_{i=1}^{n}(1+x_i)^2}$$
$$\therefore\ n \to \infty, Var(\hat{\beta}|X) \to 0,$$
$$\therefore\ \hat{\beta} \to \beta \text{ when } n \to \infty.$$

(v)

$\because \; y = (1+x)\hat{\beta} = (1+x)\frac{\sum_{i=1}^{n}(1+x_i)y_i}{\sum_{i=1}^{n}(1+x_i)^2}.$

Put $x = \bar{x}$,

$y = (1+\bar{x})\frac{\sum_{i=1}^{n}(1+x_i)y_i}{\sum_{i=1}^{n}(1+x_i)^2}$

Obviously, $y|_{x=\bar{x}} \neq \bar{y}$. Therefore, the fitted regression line doesn't pass throught $(\bar{x}, \bar{y})$.

$$\sum_{i=1}^{n}\hat{e}_i = \sum_{i=1}^{n}(y_i - \hat{y}_i)$$
$$= \sum_{i=1}^{n} y_i - \frac{[\sum_{i=1}^{n}(1+x_i)][\sum_{i=1}^{n}(1+x_i)y_i]}{\sum_{i=1}^{n}(1+x_i)^2} \neq 0$$

3. The model we are considering is with the form of: $E(W_t|H_t) = \beta_0 + \beta_1 H_t$.

(i) t-statistic=1, p-value=0.1731. The p-value is greater than α = 0.05. Thus, the null hypothesis is not rejected **at level 0.05**. It implies there is no significance effect of H_t on W_t.

(ii) The ANOVA table can be computed:

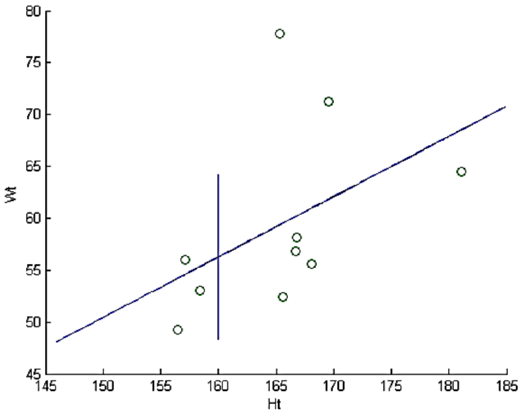
ANOVA Table

Source	df	SS	MS	F	p-value
Regression	1	159.95	159.95	2.237	0.1731
Residuals	8	572.01	71.502		
Total	9	731.96			

We are testing $H_0: \beta_0 = 0$ v.s $H_1: \beta_1 \neq 0$. F-statistic=2.237. The p-value is greater than α = 0.05. Thus, the null hypothesis is not rejected **at level 0.05**. It implies there is no significance effect of H_t on W_t.

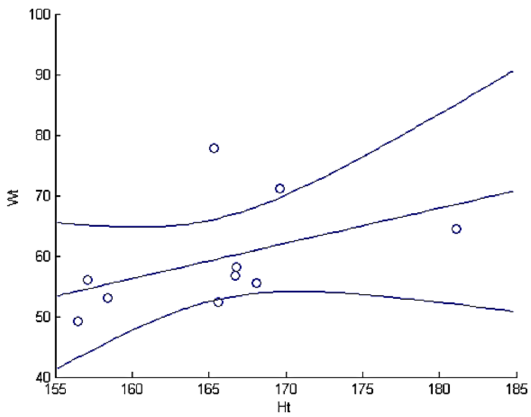
(iii) The 95% C.I. for $E(W_t|H_t = 160)$ is:

$$\hat{y} \pm t_{(8,0.025)} se(\hat{y}|x = 160) = 56.25692 \pm 2.306 \times 3.4301 = [48.3472, 64.1666].$$



(iv) The 90% C.B. for $E(W_t|H_t)$ is:

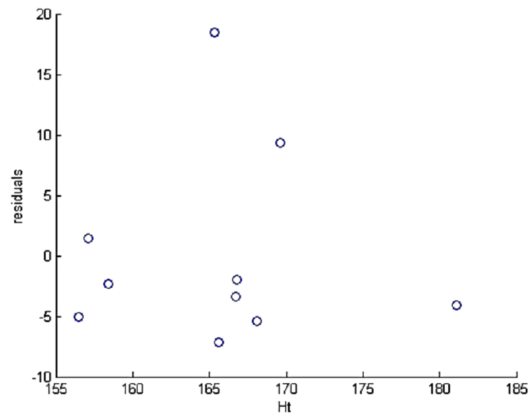
$$\widehat{\beta_0} + \widehat{\beta_1}x \pm \sqrt{2F_{(2,8,0.1)}}se(\hat{y}|x), \text{where } \widehat{\beta_0} = -36.8759, \widehat{\beta_1} = 0.5821 \; and \; F_{(2,8,0.1)} = 3.1131.$$



(v) The 99% prediction interval for a new observation at $H_t = 160$ is:

$$\tilde{y} \pm t_{(8,0.005)} se(\tilde{y}|x = 160) = [25.6423, 86.8715], \text{where } \tilde{y} = 56.2569, t_{(8,0.005)} = 3.355.$$

(vi) The residual plot seems not to be a null plot indicating a poor fit.



4.(i)

$$X(X'X)^{-1}X' = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

$$(X'X)^{-1}X'Y = \frac{Y_1 + Y_2 + Y_3}{3},$$

$$X(X'X)^{-1}X'Y = \begin{bmatrix} \frac{Y_1 + Y_2 + Y_3}{3} \\ \frac{Y_1 + Y_2 + Y_3}{3} \\ \frac{Y_1 + Y_2 + Y_3}{3} \end{bmatrix},$$

$$\bar{Y} = \frac{Y_1 + Y_2 + Y_3}{3}.$$

(ii) From the above calculation, if we just have a model with only an intercept ($E(Y|X) = \beta$), the estimate of β, which is $(X'X)^{-1}X'Y = \frac{Y_1+Y_2+Y_3}{3}$, is exactly the sample mean, which is $\bar{Y} = \frac{Y_1+Y_2+Y_3}{3}$.

(iii) $H = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$,

$Y'(I-H)Y = \frac{2Y_1^2+2Y_2^2+2Y_3-2Y_1Y_2-2Y_1Y_3-2Y_2Y_3}{3}$,

$$E(Y'(I-H)Y) = E[\frac{2Y_1^2+2Y_2^2+2Y_3-2Y_1Y_2-2Y_1Y_3-2Y_2Y_3}{3}]$$
$$= \frac{2E(Y_1^2)+2E(Y_2^2)+2E(Y_3)-2E(Y_1Y_2)-2E(Y_1Y_3)-2E(Y_2Y_3)}{3}$$
$$= \frac{2(3+2^2)+2(1+(-1)^2)+2(1+5^2)-2[-1+2(-1)]-2[0+2(5)]-2[0+(-1)(5)]}{3}$$
$$= \frac{14+4+52+6-20+10}{3} = 22$$

(iv)

$$E(Y'(I-H)Y) = E[tr(Y'(I-H)Y)]$$
$$= tr[(I-H)E(YY')]$$

$$YY' = \begin{bmatrix} Y_1^2 & Y_1Y_2 & Y_1Y_3 \\ Y_1Y_2 & Y_2^2 & Y_2Y_3 \\ Y_1Y_3 & Y_2Y_3 & Y_3^2 \end{bmatrix}$$

$$E(YY') = \begin{bmatrix} 7 & -3 & 10 \\ -3 & 2 & -5 \\ 10 & -5 & 26 \end{bmatrix}$$

$$(I-H)E(YY') = \frac{1}{3}\begin{bmatrix} 7 & -3 & -1 \\ -23 & 12 & -46 \\ 16 & -9 & 47 \end{bmatrix}$$

$$\therefore\ E(Y'(I-H)Y) = tr[(I-H)E(YY')] = \frac{7+12+47}{3} = 22$$

(v) $H = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$,

$Y'(I-H)Y = \frac{2Y_1^2+2Y_2^2+2Y_3^2-2Y_1Y_2-2Y_1Y_3-2Y_2Y_3}{3}$

$\therefore E(Y'(I-H)Y) = E\left[\frac{2Y_1^2+2Y_2^2+2Y_3^2-2Y_1Y_2-2Y_1Y_3-2Y_2Y_3}{3}\right]$

$$= \frac{2E(Y_1^2)+2E(Y_2^2)+2E(Y_3^2)-2E(Y_1Y_2)-2E(Y_1Y_3)-2E(Y_2Y_3)}{3}$$
$$= \frac{2(\sigma^2+4)+2(\sigma^2+1)+2(\sigma^2+25)-2(-2)-2(10)-2(-5)}{3}$$
$$= 2\sigma^2 + 18.$$

In another way:

$$E(Y'(I-H)Y) = E[tr(Y'(I-H)Y)]$$
$$= tr[(I-H)E(YY')]$$

$$YY' = \begin{bmatrix} Y_1^2 & Y_1Y_2 & Y_1Y_3 \\ Y_1Y_2 & Y_2^2 & Y_2Y_3 \\ Y_1Y_3 & Y_2Y_3 & Y_3^2 \end{bmatrix}$$

$$E(YY') = \begin{bmatrix} \sigma^2+4 & -2 & 10 \\ -2 & \sigma^2+1 & -5 \\ 10 & -5 & \sigma^2+25 \end{bmatrix}$$

$$(I-H)E(YY') = \begin{bmatrix} \frac{2}{3}(\sigma^2+4)+\frac{2}{3}-\frac{10}{3} & * & * \\ * & \frac{2}{3}+\frac{2}{3}(\sigma^2+1)+\frac{5}{3} & * \\ * & * & -\frac{10}{3}+\frac{5}{3}+\frac{2}{3}(\sigma^2+25) \end{bmatrix}$$

$$\therefore E(Y'(I-H)Y) = tr[(I-H)E(YY')]$$

$$= \frac{2}{3}(\sigma^2+4)+\frac{2}{3}-\frac{10}{3}+\frac{2}{3}+\frac{2}{3}(\sigma^2+1)+\frac{5}{3}-\frac{10}{3}+\frac{5}{3}+\frac{2}{3}(\sigma^2+25)$$

$$= 2\sigma^2+18.$$

5.(i)(a)

$$X = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 9 & 9 \\ 1 & 4 & 4 \\ 1 & 3 & 3 \\ 1 & 7 & 7 \\ 1 & 9 & 9 \\ 1 & 4 & 4 \\ 1 & 1 & 2 \end{bmatrix},$$

$$(X'X)^{-1} = \begin{bmatrix} 0.853 & 0.598 & -0.725 \\ 0.598 & 1.435 & -1.516 \\ -0.725 & -1.516 & 1.621 \end{bmatrix};$$

(b)

$$Y = \begin{bmatrix} 21 \\ 25 \\ 19 \\ 34 \\ 36 \\ 36 \\ 24 \\ 10 \end{bmatrix}, X'Y = \begin{bmatrix} 205 \\ 1148 \\ 1158 \end{bmatrix},$$

$$\hat{\beta} = (X'X)^{-1}X'Y = \begin{bmatrix} 21.284 \\ 13.644 \\ -12.464 \end{bmatrix};$$

(c)

$$\hat{e} = Y - X\hat{\beta} = \begin{bmatrix} -3.824 \\ -6.902 \\ -7.003 \\ 9.176 \\ 6.458 \\ 4.098 \\ -2.003 \\ 0.000 \end{bmatrix};$$

(d)

$$H = X(X'X)^{-1}X' = \begin{bmatrix} 0.29 & -0.06 & 0.23 & 0.29 & 0.06 & 0.06 & 0.24 & 0.00 \\ -0.06 & 0.41 & 0.02 & -0.06 & 0.25 & 0.41 & 0.02 & 0.00 \\ 0.23 & 0.02 & 0.20 & 0.24 & 0.09 & 0.02 & 0.20 & 0.00 \\ 0.29 & -0.06 & 0.24 & 0.29 & 0.06 & -0.06 & 0.24 & 0.00 \\ 0.06 & 0.25 & 0.09 & 0.06 & 0.19 & 0.25 & 0.09 & 0.00 \\ 0.06 & 0.41 & 0.02 & -0.06 & 0.25 & 0.41 & 0.02 & 0.00 \\ 0.24 & 0.02 & 0.20 & 0.24 & 0.09 & 0.02 & 0.20 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix};$$

(e)

$$SYY = \sum_{i=1}^{n} (y_i - \bar{y})^2 = (Y - \bar{Y})'(Y - \bar{Y}) = \left(Y - \frac{1}{n}\overrightarrow{1}\overrightarrow{1}'Y\right)' \left(Y - \frac{1}{n}\overrightarrow{1}\overrightarrow{1}'Y\right)$$

$$= Y'\left(I - \frac{1}{n}J\right)\left(I - \frac{1}{n}J\right)Y = Y'\left(I - \frac{J}{n}\right)Y = 597.875;$$

(f)

$$SSR_{eg} = Y'\left(H - \frac{J}{n}\right)Y = 339.859;$$

(g)

$$RSS = Y'(I - H)Y = 258.016;$$

(h)

$$\hat{\sigma}^2 = \frac{RSS}{n - p - 1} = \frac{258.016}{8 - 2 - 1} = 51.603;$$

(i)

$$\hat{Y} = HY = \begin{bmatrix} 24.82 \\ 31.90 \\ 26.00 \\ 24.82 \\ 29.54 \\ 31.90 \\ 26.00 \\ 10.00 \end{bmatrix};$$

(j)

$$R^2 = \frac{SSR_{eg}}{SYY} = \frac{339.859}{597.875} = 0.568;$$

(ii)

$$Var(\hat{\beta}) = \sigma^2 (X'X)^{-1} = \sigma^2 \begin{bmatrix} 0.853 & 0.598 & -0.725 \\ 0.598 & 1.435 & -1.516 \\ -0.725 & -1.516 & 1.621 \end{bmatrix};$$

Note that σ is unknown. We can estimate σ^2 by $\hat{\sigma}^2$. Hence,

$$\widehat{Var(\hat{\beta})} = 51.603 \begin{bmatrix} 0.853 & 0.598 & -0.725 \\ 0.598 & 1.435 & -1.516 \\ -0.725 & -1.516 & 1.621 \end{bmatrix};$$

(iii)Since β is constant,

$$Var(\beta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

(iv) $(n-p-1)\frac{\hat{\sigma}^2}{\sigma^2} \sim \chi^2_{n-p-1}$, $E\left[(n-p-1)\frac{\hat{\sigma}^2}{\sigma^2}\right] = n-p-1$,

$$\therefore E\,[\hat{\sigma}^2] = \sigma^2;$$

(v)

$$Var(\hat{Y}) = \sigma^2 H = \sigma^2 \begin{bmatrix} 0.29 & -0.06 & 0.23 & 0.29 & 0.06 & 0.06 & 0.24 & 0.00 \\ -0.06 & 0.41 & 0.02 & -0.06 & 0.25 & 0.41 & 0.02 & 0.00 \\ 0.23 & 0.02 & 0.20 & 0.24 & 0.09 & 0.02 & 0.20 & 0.00 \\ 0.29 & -0.06 & 0.24 & 0.29 & 0.06 & -0.06 & 0.24 & 0.00 \\ 0.06 & 0.25 & 0.09 & 0.06 & 0.19 & 0.25 & 0.09 & 0.00 \\ 0.06 & 0.41 & 0.02 & -0.06 & 0.25 & 0.41 & 0.02 & 0.00 \\ 0.24 & 0.02 & 0.20 & 0.24 & 0.09 & 0.02 & 0.20 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}; \widehat{Var(\hat{Y})} = \hat{\sigma}^2 H$$

Obviously, Y_is are independent while $\hat{Y}_i$s are dependent;

(vi)

ANOVA Table

Source	df	SS	MS	F	p-value
Regression	2	339.859	169.929	3.293	0.1223
Residuals	5	258.016	51.603		
Total	7	597.875			

$H_0: E(Y|X) = \beta_0 \leftrightarrow H_1: E(Y|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

The p-value is greater than α = 0.05. Thus, the null hypothesis is not rejected **at level 0.05**.

(vii)(a)

$$x = (1,12,-2)', \hat{y} = x'\hat{\beta} = 209.04, t_{5.0.05} = 2.015, se(\hat{y}) = \hat{\sigma}\sqrt{x'(X'X)^{-1}x} = 125.250,$$

The 90% C.I. for the fitted value is: $\hat{y} \pm t_{5,0.05}\hat{\sigma}\sqrt{x'(X'X)^{-1}x} = [-42.439,462.319];$

(b)

$$x = (1,12,-2)', \tilde{y} = x'\hat{\beta} = 209.04, t_{5,0.025} = 2.5706, se(\tilde{y}) = \hat{\sigma}\sqrt{1 + x'(X'X)^{-1}x} = 125.456,$$

The 95% C.I. for the new observation is: $\tilde{y} \pm t_{5,0.025}\hat{\sigma}\sqrt{1 + x'(X'X)^{-1}x} = [-112.557, 532.437]$;

R-code for #3:

```
x<-c(169.6,166.8,157.1,181.1,158.4,165.6,166.7,156.5,168.1,165.3)
y<-c(71.2,58.2,56,64.5,53,52.4,56.8,49.2,55.6,77.8)
model<-lm(y~x)
summary(model)
anova(model)

predict(model,data.frame(x=160),interval="confidence")
CI<-predict(model,data.frame(x=160),interval="confidence")
plot(x,y,xlim=c(150,185),ylim=c(45,80),xlab="Ht",ylab="Wt")
abline(model)
lines(c(160,160),c(CI[1,2],CI[1,3]),col="blue",type="l")

Seq<-seq(150,185,0.01)
beta0hat<-model$coefficients[1]
beta1hat<-model$coefficients[2]
Lb<-beta0hat+beta1hat*Seq-sqrt(2*qf(0.9,2,8))*8.456*sqrt((1/10+(Seq-mean(x))^2/sum((x-mean(x))^2)))
Ub<-beta0hat+beta1hat*Seq+sqrt(2*qf(0.9,2,8))*8.456*sqrt((1/10+(Seq-mean(x))^2/sum((x-mean(x))^2)))
plot(x,y,xlim=c(150,185),ylim=c(35,100),xlab="Ht",ylab="Wt")
abline(model)
points(Seq,Lb,cex=0.1,col="blue")
points(Seq,Ub,cex=0.1,col="blue")

predict(model,data.frame(x=160),level=0.99,interval="prediction")

plot(x,model$residuals,xlim=c(150,185))
```