Exercise 1.1. Find the inverse of the following matrices.

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$|A| = 4 \times 2 - 3 \times 3 = -1$.

$$\operatorname{adj}(A) = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}^T = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} \quad \therefore A^{-1} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

$|B| = 2 \times 4 \times 5 - 1 \times 4 \times 6 - 3 \times 1 \times 5 = 1$

$$\operatorname{adj}(B) = \begin{bmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{bmatrix}^T = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

Talk about the effective way of taking transpose

$$B^{-1} = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

Exercise 1.2. Find the determinant of the following matrix.

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$-$ $-$ $+$ $-$ $+$ $-$ $+$ $+$?

① Explain $\sum_{\sigma \in S_n} \prod_{i=1}^{n} A_{i,\sigma_i}$

② Count no. of elements in S_n, i.e. $n!$

③ Explain $\operatorname{sgn}(\sigma)$

$|A| = -8$, Try!

Def: $\det(A) = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} A_{i,\sigma_i}$.

where $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ & S_n is the set which includes all permutation of the set $\{1, 2, 3, \ldots, n\}$.

$\operatorname{sgn}(\sigma)$ is positive if the no. of interchanging to restore σ back to $(1, 2, \ldots, n)$ is even. Else, $\operatorname{sgn}(\sigma)$ is negative.

1.2 Basic Statistics

In this course, the mostly used concepts from your basic statistic courses are the results of transforming random variables. They are listed below.

Theorem 1.3. *Let $X \sim \mathcal{N}(\mu, \sigma^2)$, then*

$$\left(\frac{X-\mu}{\sigma}\right)^2 \sim \chi^2(1)$$

Theorem 1.4. *Let $V_1, V_2, \ldots, V_n$ be i.i.d. random variables where $V_i \sim \chi^2(k_i), i = 1, 2, \ldots, n$, then*

$$V_1 + V_2 + \cdots + V_n \sim \chi^2\left(\sum_{i=1}^{n} k_i\right)$$

Theorem 1.5. *Let $Z \sim (N)(0, 1)$ and $V \sim \chi^2(k)$ be independent, then*

Typo

$$T = \frac{Z}{\sqrt{V/k}} \sim t(k)$$

Theorem 1.6. *Let $V_1 \sim \chi^2(k_1)$ and $V_2 \sim \chi^2(k_2)$, then*

$$F = \frac{V_1/k_1}{V_2/k_2} \sim F(k_1, k_2)$$

In statistics, tests and estimations are often based on the above theorems, and STAT3008 is no exception. You may find plenty of application of them in the construction of confidence intervals and tests very soon.

Exercise 1.3. Prove Theorem 1.4.

Let $V = V_1 + V_2 + \ldots + V_n$, $\mathbb{E}(e^{tV}) = \mathbb{E}(e^{t(V_1+V_2+\ldots+V_n)})$

$= \mathbb{E}(e^{tV_1})\mathbb{E}(e^{tV_2})\ldots\mathbb{E}(e^{tV_n})$ (by independence)

$= (1-2t)^{-\frac{k_1}{2}}(1-2t)^{-\frac{k_2}{2}}\ldots(1-2t)^{-\frac{k_n}{2}}$

$= (1-2t)^{-\frac{1}{2}\sum_{i=1}^{n} k_i}$ By uniqueness of m.g.f., $V \sim \chi^2(\sum_{i=1}^{n} k_i)$

Exercise 1.4. Provide an example of hypothesis testing and explain the philosophy of a test.

e.g. Test of Mean with Known Variance

$Y_1 \ldots Y_n$ i.i.d., $Var(Y_i) = \sigma^2$, $\mathbb{E}(Y_i) = \mu$

$\bar{Y} \sim N(\mu, \sigma^2)$ asymptotically

$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$

Test stat: $z = \frac{\bar{X} - \mu}{\sigma} \sim N(0, 1)$

asymptotically, by CLT.

For 95% confidence, H_0 is rejected if $|z| > 1.96$

* Under H_0, we have 95% confidence that $|z| \le 1.96$.

* If $|z| > 1.96$, ① It has only 5% prob in H_0

② The result is more possible in H_1.

$\Rightarrow$ reject H_0 rejection depends on H_1

GIVE A GENERAL DESCRIPTION.