STAT3008 Exercise 7 Solutions

(2011-2012 2nd Semester)

Q1. (4.3)

4.3.1

Since A and D are exact linear combinations of T_1 and T_2, only two of the four terms added after the intercept can be estimated.

4.3.2

The intercept, $\widehat{\sigma^2}$ and R^2 are the same within the 4 models. The estimates for T_1 and T_2 are exactly the same in M1 and M4 because M4 is completely the same as M1 after deleting the aliased variables. Estimate for T_2 in M3 is different with those in M1 and M4. Also, estimate of D in M2 is different with that in M3.

4.3.3

In M1, $\frac{\partial E(Y \mid T_1 = t_1, T_2)}{\partial T_2} = \beta_{21}$. In M3, $\frac{\partial E(Y \mid T_1, D = t_1 - t_2)}{\partial T_2} = \beta_{23}$. Therefore, the estimate in M1 is the change in Y for a unit increase in T_2 for a given value of T_1. However, the estimate in M3 is the change of Y for a unit increase in T_2 for a given value of $D = t_1 - t_2$.

Q2. (4.4)

4.4.1

$$E(\log(Y) \mid X = x) = \beta_0 + \beta_1 \log x$$
$$E(Y \mid X = x) \approx \exp(\beta_0 + \beta_1 \log x) = x^{\beta_1} e^{\beta_0}$$

$$\frac{\partial E(Y \mid X = x)}{\partial x} = \beta_1 e^{\beta_0} x^{\beta_1 - 1} = \frac{\beta_1}{x} x^{\beta_1} e^{\beta_0} = \frac{\beta_1}{x} E(Y \mid X = x)$$
$$\frac{\frac{\partial E(Y \mid X = x)}{\partial x}}{E(Y \mid X = x)} = \frac{\beta_1}{x}$$

Therefore, the rate of change per uit of Y decreases inversely with x.

4.4.2

Changing the base of logs would multiply the equations by a constant only. The rate of change per unit of Y won't change. Thus, the value of β_1 won't change.

Q3. (4.7)

$E(Y \mid X_1 = x_1, X_2 = x_2) = 3 + 4x_1 + 2x_2$

$E(Y \mid X_1 = x_1) = 3 + 4x_1 + 2E(X_2 \mid X_1 = x_1)$

Therefore, the above expectation is linear if $E(X_2 \mid X_1 = x_1) = a + bx_1$.

$E(Y \mid X_1 = x_1) = 3 + 4x_1 + 2(a + bx_1)$

$= 3 + 2a + (4 + 2b)x_1$

And the coefficient of x_1 is negative if $4 + 2b < 0 \rightarrow b < -2$.

Q4. (4.8)

4.8.1

$$Sex = \begin{cases} 1 & if \quad Female \\ 0 & if \quad Male \end{cases}$$

$E(Salary \mid Male) = 24697$, i.e. the expected salary for a male faculty member is $24697.

$E(Salary \mid Female) = 24697 - 3340 = 21357$, i.e. the expected salary for a female faculty member is $21357, which is $3340 lower than male's.

4.8.2

$E(\widehat{Salary} \mid Sex) = 18065 + 201Sex + 759E(Years \mid Sex)$

$But \qquad E(\widehat{Salary} \mid Sex) = 24697 - 3340Sex$

$\Rightarrow 24697 - 3340Sex = 18065 + 201Sex + 759E(Years \mid Sex)$

$$\Rightarrow E(Years \mid Sex) = \frac{6632 - 3541Sex}{759}$$

$E(Years \mid Male) = 8.7, E(Years \mid Female) = 4.0$

Therefore, a male faculty member seems to have more work experience than female does. It is consistent.